Box-Ball Systems from Integrable Systems and Probabilistic Perspectives

CRM Workshop, 19-23 September 2022

Centre de Recherches Mathématiques, Université de Montréal

Organisers: David Croydon (Kyoto University), Makiko Sasada (University of Tokyo), Satoshi Tsujimoto (Kyoto University)

Monday 19th September	Tuesday 20th September	Wednesday 21st September	Thursday 22nd September	Friday 23rd September
10:00–10:50 Takahashi*	10:00–10:50 Tsujimoto	10:00–10:50 Tsujimoto	10:00–10:50 Kuniba*	10:00–10:50 Kuniba*
11:00–11:50 Ferrari*	11:00–11:50 Ferrari*	11:00–11:50 Sasada	11:00–11:50 Sasada	11:00–11:50 Croydon
13:30–14:20 Okado	13:30–14:20 O'Connell*		13:30–14:20 Willox	13:00–13:50 Doyon
14:30–15:20 Suda	14:30–15:20 Quastel	Free discussion	14:30–15:20 Lyu	14:00–14:50 Spohn*
15:40–16:30 Maeda	15:40–16:30 Sasamoto*		15:40–16:30 Wang	15:10–16:00 Pasquier
	16:40–17:30 Corwin		16:40–17:30 Franceschini	

∗ = online participant

Morning lectures

Daisuke Takahashi* ..... *Viewpoints brought by the box and ball system*
Pablo Ferrari* ..... *Title*
Satoshi Tsujimoto ..... *Discrete integrable systems and their ultradiscretizations*
Makiko Sasada ..... *Bi-infinite dynamics and stationary measures for discrete KdV- and Toda-type systems*
Atsuo Kuniba* ..... *Integrability of box-ball systems and randomized box-ball systems*
David Croydon ..... *Generalized hydrodynamic limit for the box-ball system*

Afternoon talks

Masato Okado ..... *Box-ball systems: reviews and recent developments*
Hayate Suda .. *Relationships between two linearizations of the box-ball system: rigged configuration and slot configuration*
Kazuki Maeda ..... *Box-ball systems and biorthogonal polynomials*
Neil O'Connell* ..... *Toda and probability*
Jeremy Quastel ..... *Polynuclear growth and the Toda lattice*
Tomohiro Sasamoto* ..... *BBS like behaviors in skew RSK dynamics*
Ivan Corwin ..... *Invariant measures for KPZ systems with boundaries*
Ralph Willox ..... *Direct and inverse scattering for the ultradiscrete KdV equation*
Hanbaek Lyu ..... *Scaling limit of soliton statistics of a multicolor box-ball system*
Minmin Wang ..... *Soliton speeds in the box-ball system*
Chiara Franceschini ..... *Some results for hard rods of inhomogeneous size*
Benjamin Doyon ..... *Title*
Herbert Spohn* ..... *Spacetime correlations of the classical Toda chain in thermal equilibrium*
Vincent Pasquier ..... *Hydrodynamics of the box-ball system*

`http://www.crm.umontreal.ca/2022/BBS22/index_e.php`

Morning lectures

Daisuke Takahashi* (Waseda University)

Viewpoints brought by the box and ball system

Abstract

Pablo Ferrari* (Universidad de Buenos Aires)

Title

Abstract

Satoshi Tsujimoto (Kyoto University)

Discrete integrable systems and their ultradiscretizations

Abstract

Makiko Sasada (University of Tokyo)

Bi-infinite dynamics and stationary measures for discrete KdV- and Toda-type systems

1. Bi-infinite dynamics for KdV- and Toda-type discrete integrable systems based on generalized Pitman's transformation

Abstract

2. Stationary measures for discrete KdV- and Toda-type systems

Abstract

Atsuo Kuniba* (University of Tokyo)

Integrability of box-ball systems and randomized box-ball systems

1. Integrability of box-ball systems: crystals, Bethe ansatz and ultradiscretization

Abstract

2. Randomized box-ball systems: density plateaux, current correlations and large deviations

Abstract

David Croydon (Kyoto University)

Generalized hydrodynamic limit for the box-ball system

Abstract

Afternoon talks

Masato Okado (Osaka City University)

Box-ball systems: reviews and recent developments

Abstract

Hayate Suda (Keio University)

Relationships between two linearizations of the box-ball system: rigged configuration and slot configuration

Abstract

Kazuki Maeda (University of Fukuchiyama)

Box-ball systems and biorthogonal polynomials

Abstract

Neil O'Connell* (University College Dublin)

Toda and probability

Abstract

Jeremy Quastel (University of Toronto)

Polynuclear growth and the Toda lattice

The polynuclear growth model is one of the most important models in the KPZ universality class. Generally it has been studied in the droplet geometry, where it is equivalent to the longest increasing subsequence of a random permutation, whose solution sparked the KPZ revolution. We study it for general initial data and show that it is an integrable Markov process sharing the key structures of the KPZ fixed point, determinantal formulas for the transition probabilities and fixed time n-point distributions governed by completely integrable equations, the non-Abelian 2D Toda lattice. Joint with Konstantin Matetski and Daniel Remenik.

Tomohiro Sasamoto* (Tokyo Institute of Technology)

BBS like behaviors in skew RSK dynamics

Abstract

Ivan Corwin (Columbia University)

Invariant measures for KPZ systems with boundaries

Abstract

Ralph Willox (University of Tokyo)

Direct and inverse scattering for the ultradiscrete KdV equation

In this talk I will describe a method for solving the initial value problem for the ultradiscrete KdV (udKdV) equation over the real numbers, which includes the initial value problem for the famous Takahashi-Satsuma Box & Ball system as a special case. The method is an ultradiscrete analogue of the inverse scattering transform for the usual, continuous, KdV equation.

In particular, I will first explain how to obtain the solution to a direct scattering problem for the udKdV equation for any potential, over the real numbers, with compact support, by explicitly constructing bound state and non-bound state eigenfunctions. I will then show how to reconstruct the potential in the scattering problem at different time steps which satisfy the udKdV evolution, using an ultradiscrete analogue of a Darboux dressing transformation. This reconstruction is based on data that uniquely characterise the soliton content and the so-called 'background' part in the initial potential, data which are obtained from the initial potential by successive Darboux undressing transformations.

This talk is based on the paper: "Darboux dressing and undressing for the ultradiscrete KdV equation" J.J.C. Nimmo, C.R. Gilson and R. Willox, J. Phys. A: Math. Theor. 52 (2019) 445201 (36pp).

Hanbaek Lyu (University of Wisconsin – Madison)

Scaling limit of soliton statistics of a multicolor box-ball system

The box-ball systems (BBS) are integrable cellular automata whose long-time behavior is characterized by the soliton solutions, and have rich connections to other integrable systems such as Korteweg-de Veris equation. Probabilistic analysis of BBS is an emerging topic in the field of integrable probability, which often reveals novel connection between the rich integrable structure of BBS and probabilistic phenomena such as phase transition and invariant measures. In this talk, we give an overview on the recent development in scaling limit theory of the classical Takahashi-Satsuma BBS as well as the multicolor BBS with one-sided random initial configurations. The integrability of BBS in this setting allows one to read-off the final soliton statistics directly from the initial configuration through various combinatorial operations. For the Takahashi-Satsuma case, these include hill-flattening operations of carrier process for soliton numbers and Pitman's water level process for soliton lengths. For the multicolor case, we use a modified Greene-Kleitman invariants for BBS, circular exclusion processes, Kerov–Kirillov–Reshetikhin bijection, combinatorial R, and Thermodynamic Bethe Ansatz to extract the corresponding soliton statistics.

Minmin Wang (University of Sussex)

Soliton speeds in the box-ball system

In this talk, I'll focus on the system of linear equations for the effective soliton speeds in the box-ball system that has been obtained in [Ferrari, Nguyen, Rolla and Wang 2021]. I'll discuss the soliton dynamics that give rise to these equations as well as the mathematical tools that can turn the intuitive picture into rigorous arguments. Based on joint work with Pablo Ferrari, Chi Nguyen and Leo Rolla.

Chiara Franceschini (University of Modena)

Some results for hard rods of inhomogeneous size

Abstract

Benjamin Doyon (King's College London)

Title

Abstract

Herbert Spohn* (Technical University Munich)

Spacetime correlations of the classical Toda chain in thermal equilibrium

For integrable many-body systems one expects that, in leading order, equilibrium spacetime correlations scale ballistically with smooth shape functions. We report on novel results for the classical Toda lattice. (i) Correlations are obtained microscopically through molecular dynamics simulations over a wide range of equilibrium paramentes. (ii) These results are compared with predictions from the Landau-Lifshitz theory based on generalized hydrodynamics.

Vincent Pasquier (Université Paris Saclay)

Hydrodynamics of the box-ball system

Abstract