China-Japan Joint Workshop on Integrable Systems 2019

Shonan Village Center, Hayama, Kanagawa, Japan

August 18-23, 2019

China-Japan Joint Workshop on Integrable Systems 2019

Date: August 18-23, 2019
Venue: Shonan Village Center

Executive Committee
China Side
Yong Chen (China East Normal University), Engui Fan (Fudan University), Xianguo Geng (Zhengzhou University), Jingsong He (Ningbo University), Xingbiao Hu (AMSS, Chinese Academy of Sciences), Qingping Liu (China University of Mining), Senyue Lou (Ningbo University), Changzheng Qu (Ningbo University), Lixin Tian (Nanjing Normal University), Zhenya Yan (AMSS, Chinese Academy of Sciences), Ruoxia Yao (Shannxi Normal University), Dajun Zhang (Shanghai University), Youjin Zhang (Tsinghua University), Ruguang Zhou (Jiangsu Normal University), Zuonong Zhu (Shanghai Jiaotong University), Dafeng Zuo (University of Science and Technology of China)

Japan Side
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Local Organizing Committee
Daisuke Takahashi (Waseda University), Ken-ichi Maruno (Waseda University), Kouichi Toda (Toyama Prefectural University), Hidetomo Nagai (Tokai University), Linyu Peng (Waseda University), Kazushige Endo (Waseda University)

The workshop is supported by JST CREST.

Information

Conference room: Auditorium (Conference Floor)
Poster Session: Foyer (the space next to Auditorium)
Breakfast, Lunch, Dinner: Cafeteria Oak (1F)
Banquet (August 20): Foyer (the space next to Auditorium)

Breakfast: 7:30-9:00
Lunch: 11:30-13:30
Dinner: 18:00-19:30 (Last Order)

Wi-Fi connection is available. You can find the information of Wi-Fi connectipn in your room.

Poster Session: Please put your poster on poster board in August 19 and remove your poster in the morning of August 21.

All guest rooms, conference rooms and restaurantes at Shonan Village Center are non-smoking. There are smoking room in Conference Fllor and Pocket Lounge in 2F, 3F.

Shop (1F): 8:00-22:30
Smoking area: Pocket Lounge (2F, 3F)
Laundry Room 1F, 2F
Aqua Club (Pool): 7:00-22:00

There is Business Room in Conference Floor. There are PCs and a photocoier.

If you want to have discussion with participants at night, you can use Lounge LEAF in Lobby (after 5pm).

Convinience Store: There is a convinience store (Family Mart) near Shonan Village Center. The distance from Shonan Village Center is 700 meters (7 minutes walk).

Address: Shonan Village Center, 1560-39 Kamiyamaguchi, Hayama Miura-gun, Kanagawa 240-0198
TEL: +81-46-855-1800

Payment: Please pay accommodation fee at Front Desk during the workshop.

List of Participants

China Side

Xiang-Ke Chang	Chinese Academy of Sciences
Kui Chen	Fudan University
Yong Chen	East China Normal University
En-Gui Fan	Fudan University
Xing-Biao Hu	Chinese Academy of Sciences
Man Jia	Ningbo University
Chun-Xia Li	Capital Normal University
Qing-Ping Liu	China University of Mining and Technology (Beijing)
Xiao-Chuan Liu	Xian JiaoTong University
Sen-Yue Lou	Ningbo University
Li-Yuan Ma	Zhejiang University of Technology
Shou-Feng Shen	Zhejiang University of Technology
Jian-Qing Sun	Ocean University of China
Xiao-Yan Tang	East China Normal University
Chao-Zhong Wu	Sun Yat-Sen University
Zhi-Wei Wu	Sun Yat-Sen University
Jian Xu	University of Shanghai for Science and Technology
Di Yang	University of Science and Technology of China
Ruo-Xia Yao	Shaan Xi Normal University
Guo-Fu Yu	Shanghai JiaoTong University
Cheng Zhang	Shanghai University
Da-Jun Zhang	Shanghai University
Hai-Qiong Zhao	Shanghai University of International Business and
Ru-Guang Zhou	Jiangsu Normal University
Zuo-Nong Zhu	Shanghai JiaoTong University
Da-Feng Zuo	University of Science and Technology of China

Japan Side

Kanae Akaiwa	Kyoto Sangyo University
Kazushige Endo	Waseda University
Keisuke Hosaka	Waseda University
Shin Isojima	Hosei University
Saburo Kakei	Rikkyo University
Shuhei Kamioka	Kyoto University
Katsuki Kobayashi	Kyoto University
Takeo Kojima	Yamagata University
Yu Luo	Kyoto University
Kazuki Maeda	University of Fukuchiyama
Kenichi Maruno	Waseda University
Keisuke Matsuya	Musashino University
Hiroshi Miki	Meteorological College
Atsushi Nagai	Tsuda University
Hidetomo Nagai	Tokai University
Hajime Nagoya	Kanazawa University
Nobutaka Nakazono	Tokyo University of Agriculture and Technology
Yuki Nishida	Doshisha University
Yasuhiro Ohta	Kobe University
Ryotaro Oshima	Waseda University
Linyu Peng	Waseda University
Takao Suzuki	Kindai University
Daisuke Takahashi	Waseda University
Kanesaki Takasaki	Kindai University
Yuta Tanaka	Waseda University
Kouichi Toda	Toyama Prefectual University
Satoshi Tsujimoto	Kyoto University
Yoshihide Watanabe	Doshisha University
Ralph Willox	the University of Tokyo
Juntaro Yamamoto	Waseda University
Hideshi Yamane	Kuwansei Gakuin University
Haoyu Yang	Waseda University

China-Japan Joint Workshop on Integrable Systems 2019

Date : August 18-23, 2019
Venue : Auditorium, Shonan Village Center, Hayama, Miura-gun, Kanagawa, Japan

August 18	Arrival & Registration (15:00-18:00)
18:00-20:00	Dinner (Cafeteria OAK)
20:00-23:00	Free Discussion
August 19	
7:30-9:00	Breakfast (Cafeteria OAK)
9:00-10:40	Free Discussion & Registration
10:40-10:55	Opening, Daisuke Takahashi (Waseda University)
10:55-11:20	Sen-Yue Lou (Ningbo University) Multi-place systems and multi-place physics
11:20-11:45	Satoshi Tsujimoto (Kyoto University) On classical orthogonal polynomials from Jacobi to -1 polynomials
11:45-12:10	Ruoxia Yao (ShaanXi Normal University) On a nonlocal Alice-Bob system of nonlinear Schrödinger equation: Bilinear Bäcklund transformation, Darboux transformation and soliton solutions
	Lunch (Cafeteria OAK)
13:40-14:05	Yu Luo (Kyoto University) Exceptional $q = -1$ polynomials
14:05-14:30	Xiangke Chang (Chinese Academy of Sciences) Isospectral flows related to Frobenius-Stickelberger-Thiele polynomials
14:30-14:55	Hiroshi Miki (Meteorological College) Quantum walks on graphs associated with bivariate Krawtchouk polynomials
14:55-15:20	Yong Chen (East China Normal University) Constructing two-dimensional optimal system of the group invariant solutions
	Coffee Break
15:40-16:05	Jianqing Sun (Ocean University of China) A numerical study of the 3-periodic wave solutions to KdV-type equations
16:05-16:30	Shin Isojima (Hosei University) Ultradiscretization with parity variables of the hard-spring equation and its conserved quantity
16:30-16:55	Ruguang Zhou (Jiangsu Normal University) Permutation matrices and multi-component coupled integrable lattice systems
16:55-17:20	Hidetomo Nagai (Tokai University) Discrete and ultradiscrete mixed soliton solutions
17:20-17:45	Zuonong Zhu (Shanghai Jiao Tong University) On A Reverse Space-Time Nonlocal Sasa-Satsuma Equation
18:00-19:30	Dinner (Cafeteria OAK)
19:30-23:00	Free Discussion

August 20

7:30-9:00	Breakfast (Cafeteria OAK)
9:10-10:20	Short Communication (10 min.) Keisuke Hosaka, Yu Luo, Liyuan Ma Ryotaro Oshima, Yuta Tanaka, Haoyu Yang
	Coffee Break
10:40-11:05	Kanehisa Takasaki (Kindai University) Volterra-type hierarchies for specialized hypergeometric tau functions
11:05-11:30	Dafeng Zuo (University of Science and Technology of China) Extended affine Weyl groups of BCD-type: their Frobenius manifolds and Landau–Ginzburg superpotentials
11:30-11:55	Hajime Nagoya (Kanazawa University) Determinant formulas for tau functions of q-Painlevé systems in terms of q-Nekrasov partition functions
	Lunch (Cafeteria OAK)
13:30-13:55	Dajun Zhang (Shanghai University) On multidimensional consistency
13:55-14:20	Nobutaka Nakazono (Tokyo University of Agriculture and Technology) Consistency around a cuboctahedron
14:20-14:45	Chun-Xia Li (Capital Normal University) Gauge transformations for the twisted derivation and quasideterminant solutions to the super KdV equation
14:45-15:10	Takeo Kojima (Yamagata University) Wakimoto realization of the quantum affine superalgebra $U_q(\widehat{sl}(M\|N))$

Coffee Break

15:20-17:00	Poster Session Kui Chen, Engui Fan, Keisuke Hosaka, Xing-Biao Hu, Liyuan Ma Ryotaro Oshima, Shou-Feng Shen, Yuta Tanaka, Xiaoyan Tang, Haoyu Yang
18:30-20:30	Banquet (Foyer, next to Auditorium)
20:00-23:00	Free Discussion

August 21

7:30-9:00	Breakfast (Cafeteria OAK)
9:10-9:35	Linyu Peng (Waseda University) A general prolongation formulation for symmetries of differential-difference equations
9:35-10:00	Xiao-Chuan Liu (Xian Jiao Tong University) Orbital stability of peaked solitary waves with arbitrary order of nonlinearities
	Coffee Break
10:20-10:45	Atsushi Nagai (Tsuda University) The best constant of discrete Sobolev inequalities
10:45-11:10	Jian Xu (University of Shanghai for Science and Technology) Riemann-Hilbert approach for the integrable modified Camassa-Holm equation with cubic nonlinearity
11:10-11:35	Saburo Kakei (Rikkyo University) Algebro-geometric solutions of lattice soliton equations arising from Toda lattice hierarchy
11:35-12:00	Zhiwei Wu (SunYat-Sen University) Darboux transforms for KdV-type hierarchies associated to affine Kac-Moody algebra of B-type
	Lunch (Cafeteria OAK)
13:30-13:55	Takao Suzuki (Kindai University) Cluster algebra and q-Painlevé equations
13:55-14:20	Guo-Fu Yu (Shanghai Jiao Tong University) Invariant subspaces of biconfluent Heun operators and special solutions of Painlevé IV
14:20-14:45	Kanae Akaiwa (Kyoto Sangyo University) Construction of Laurent-Jacobi matrices with prescribed eigenvalues via orthogonal polynomials
14:45-15:10	Chaozhong Wu (SunYat-Sen University) Virasoro symmetries for Drinfeld-Sokolov hierarchies and equations of Painlevé type
15:10-15:35	Shuhei Kamioka (Kyoto University) The discrete Toda equation proves the Aztec diamond theorem
	Coffee Break
15:55-16:20	Yuki Nishida (Doshisha University) A fuzzification scheme for three states cellular automata
16:20-16:45	Di Yang (University of Science and Technology of China) On quasi-triviality of a class of PDEs
16:45-17:10	Kazuki Maeda (The University of Fukuchiyama) Generalized box–ball systems and nonautonomous discrete Toda lattices
18:00-19:30	Dinner (Cafeteria OAK)
19:30-23:00	Free Discussion

August 22

7:30-9:00	Breakfast (Cafeteria OAK)
9:00-9:25	Kazushige Endo (Waseda University) Common property of asymptotic behavior of some probabilistic cellular automata
9:25-9:50	Cheng Zhang (Shanghai University) Integrable mappings arising from open boundary reductions
	Coffee Break
10:10-10:35	Keisuke Matsuya (Musashino University) Spatial pattern of ultradiscretizable discrete Gray-Scott model and Turing instabilities of its equilibrium solutions
10:35-11:00	Qingping Liu (China University of Mining and Technology) Multi-soliton solutions of the two-component Camassa-Holm system: Darboux transformation approach
11:00-11:25	Ralph Willox (the University of Tokyo) On the direct and inverse scattering problems for udKdV
11:25-11:35	Closing
	Lunch (Cafeteria OAK)
13:00-17:30	Free Discussion
17:30-20:00	Dinner (Minemoto Honten in Kamakura)
20:00-23:00	Free Discussion
August 23	Free Discussion
10:00	Departure

Poster Session (in August 20):

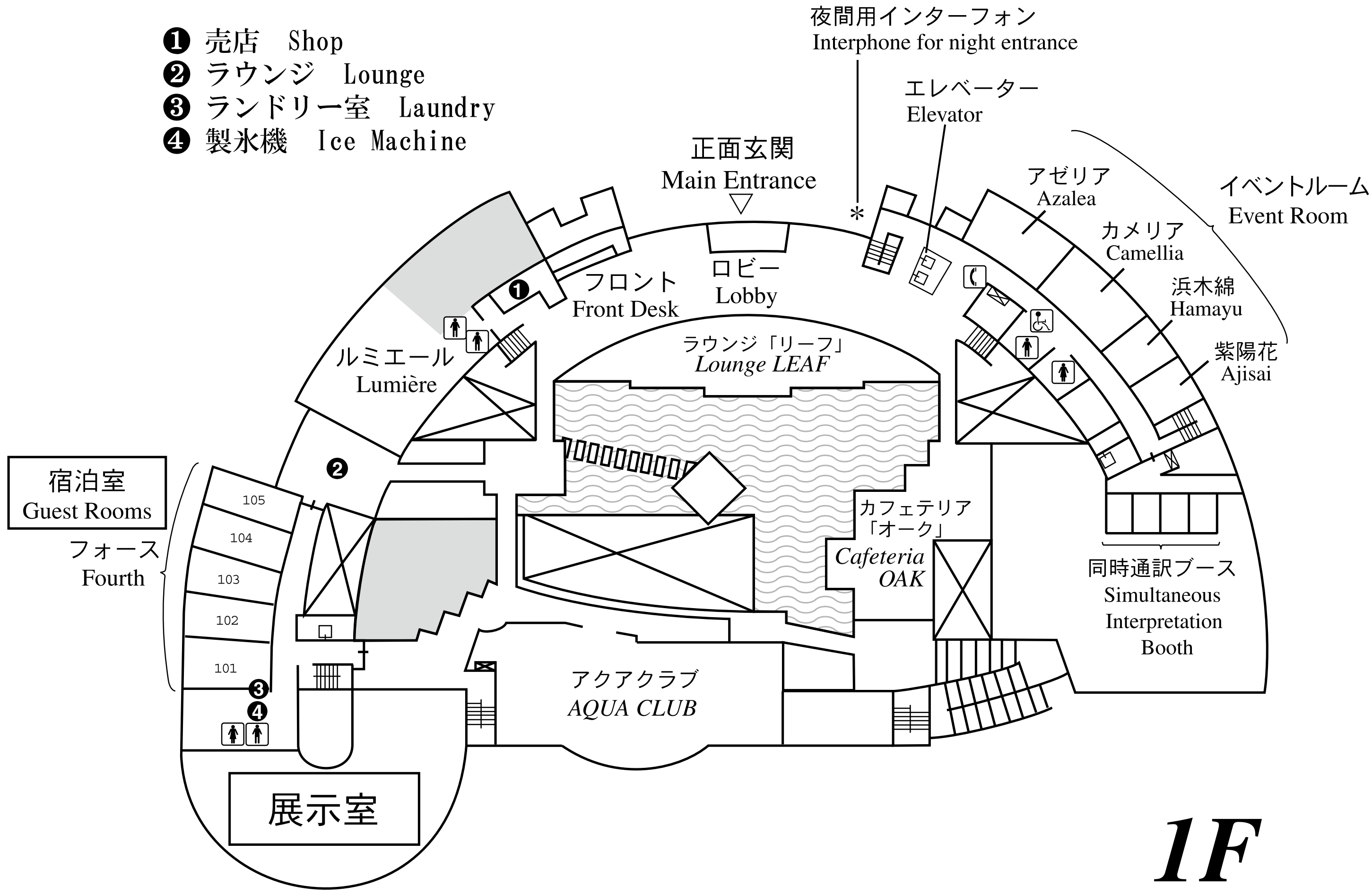
Kui Chen (Fudan Univ.): Bilinear equations and solutions of k-constrained DΔKP
Engui Fan (Fudan Univ.): Riemann-Hilbert Approach to Orthogonal Polynomials and Random Matrices
Keisuke Hosaka (Waseda Univ.): Initial value problem of max equation with an external variable and of a system of max-equations
Xing-Biao Hu (Chinese Acad. of Sci.): Convergence acceleration algorithms and discrete integrable systems
Liyuan Ma (Zhejiang Univ. of Technol.): Abundant exact solutions to the discrete complex mKdV equation by Darboux transformation
Ryotaro Oshima (Waseda Univ.): C program to solve the initial value problem of max-min equations
Shou-Feng Shen (Zhejiang Univ. of Technol.): Completion of the Ablowitz-Kaup-Newell-Segur integrable coupling
Yuta Tanaka (Waseda Univ.): Soliton solutions of the DKP equation and networks
Xiaoyan Tang (East China Normal Univ.): A general nonlocal nonlinear Schrödinger equation with shifted parity, charge-conjugate and delayed time reversal
Haoyu Yang (Waseda Univ.): Construction of exact soliton solutions in the spinor $F = 1$ Bose-Einstein condensates by Hirota bilinear method

Local Organizing Committee:
Daisuke Takahashi (Waseda Univ.), Ken-ichi Maruno (Waseda Univ.), Kouichi Toda (Toyama Prefectural Univ.), Hidetomo Nagai (Tokai Univ.), Linyu Peng (Waseda Univ.), Kazushige Endo (Waseda Univ.)

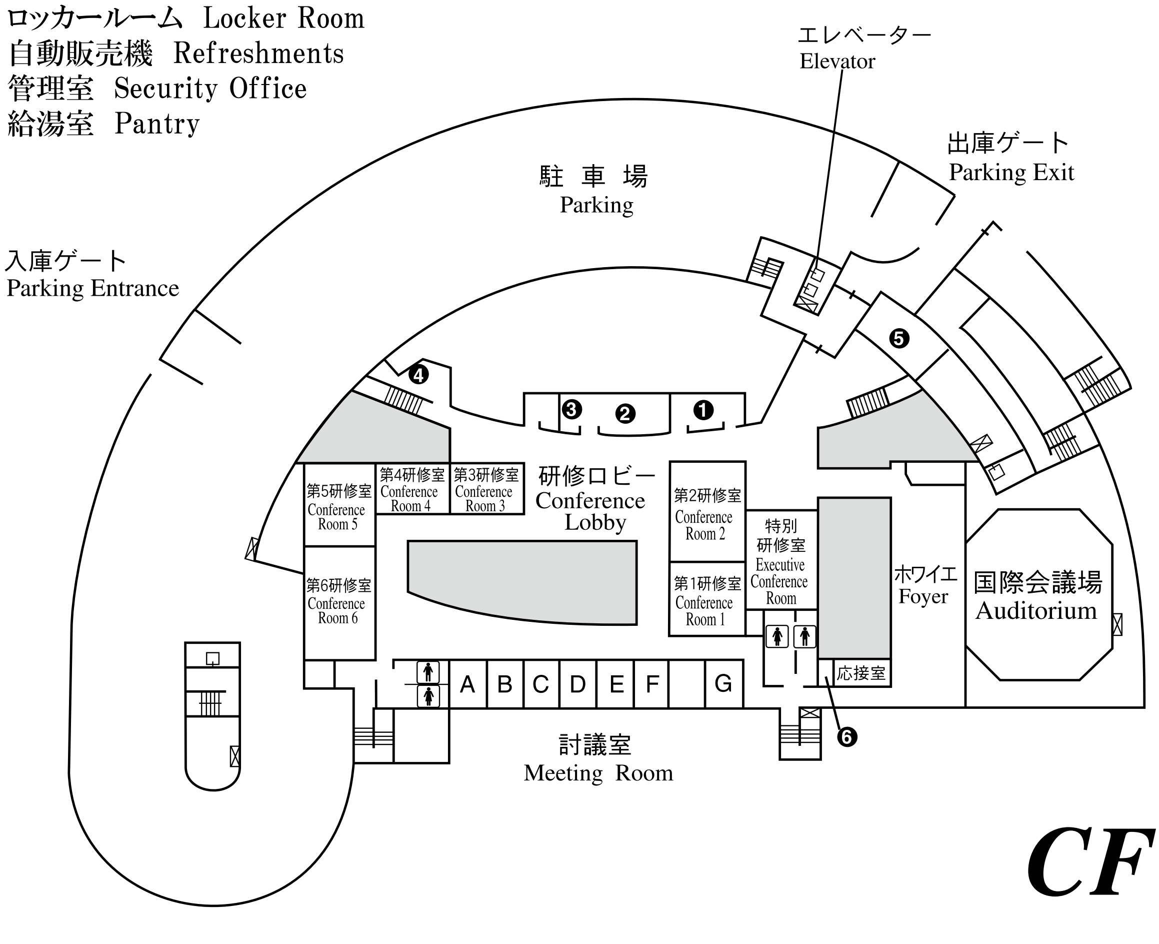
This workshop was supported by JST CREST.

館内マップ

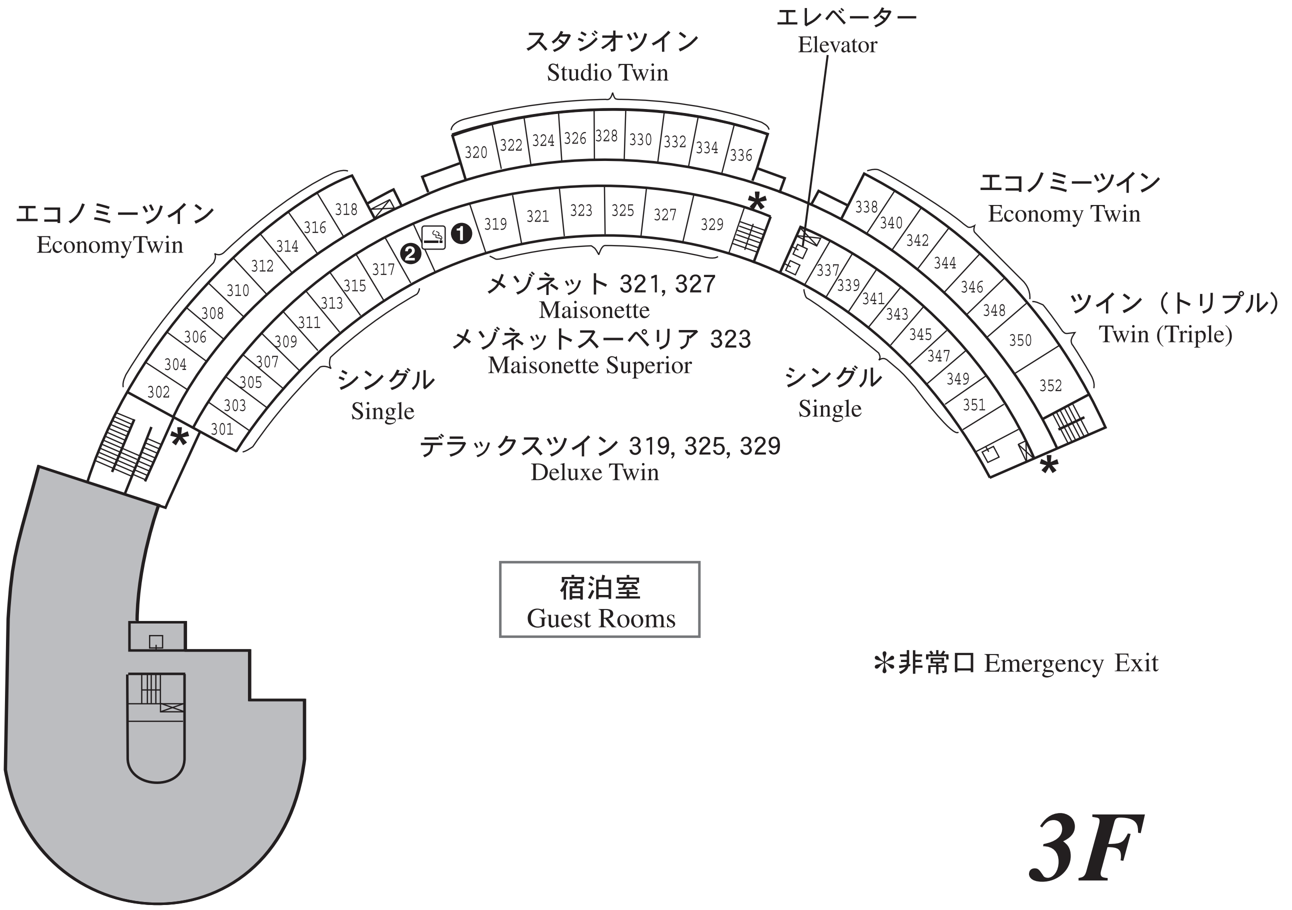
❶ 売店　Shop
❷ ラウンジ　Lounge
❸ ランドリー室　Laundry
❹ 製氷機　Ice Machine



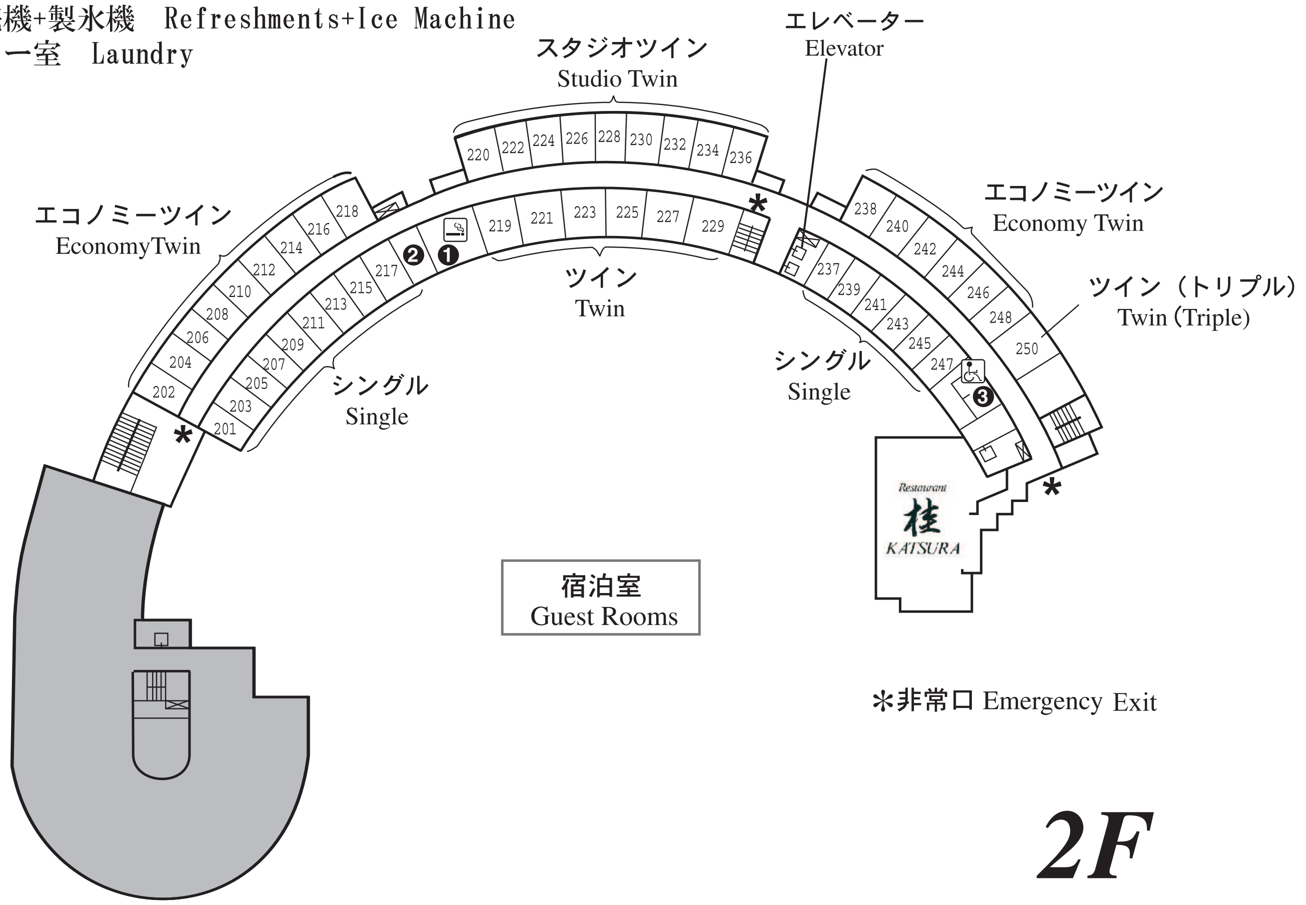
❶ ビジネスルーム　Business Room
❷ 喫煙ルーム　Smoking Room
❸ ロッカールーム　Locker Room
❹ 自動販売機　Refreshments
❺ 管理室　Security Office
❻ 給湯室　Pantry



❶ ポケットラウンジ　Pocket Lounge
❷ 自動販売機+製氷機　Refreshments+Ice Machine

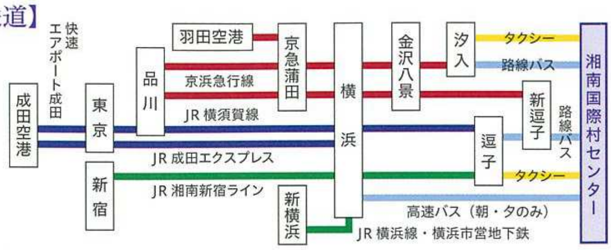


❶ ポケットラウンジ　Pocket Lounge
❷ 自動販売機+製氷機　Refreshments+Ice Machine
❸ ランドリー室　Laundry



交通案内 TRANSPORTATION

【鉄道】



【車】

【鉄道】TRAIN

JR 逗子駅までの所要時間　Time for JR Zushi Station

東京駅または新宿駅から 約 60 分
JR 成田空港駅から　　　約 2 時間
※快速エアポート成田利用なら乗換なし　約 2.5 時間
京急羽田空港駅から　　約 60 分
新横浜駅から　　　　　約 50 分

60 minutes from Tokyo or Shinjuku
2 hours from Narita Airport by Narita Express
*by Airport Narita (Rapid Service), no need to change the train (2.5hours)
60 minutes from Haneda Airport
50 minutes from Shin Yokohama

【バス】BUS

JR 逗子駅前又は京急新逗子駅前より

「(逗 16) 葉山大道～湘南国際村センター」行き利用で約 30 分
「(逗 26) 三浦半島中央道路～湘南国際村センター」行き利用で約 20 分
京急汐入駅前より「湘南国際村（汐 16）」行き利用で約 34 分
「湘南国際村センター前」下車
横浜駅（YCAT）より「湘南国際村センター」行き京急高速バス利用で約 45 分

30 minutes from JR Zushi Station by local bus line 逗 16 to Shonan Village Center
20 minutes from JR Zushi Station by local bus line 逗 26 to Shonan Village Center
34 minutes from Keikyu Shioiri Station by local bus line 汐 16 to Shonan Village Center
45 minutes from Yokohama Station (YCAT) by highway bus to Shonan Village Center

【タクシー】TAXI

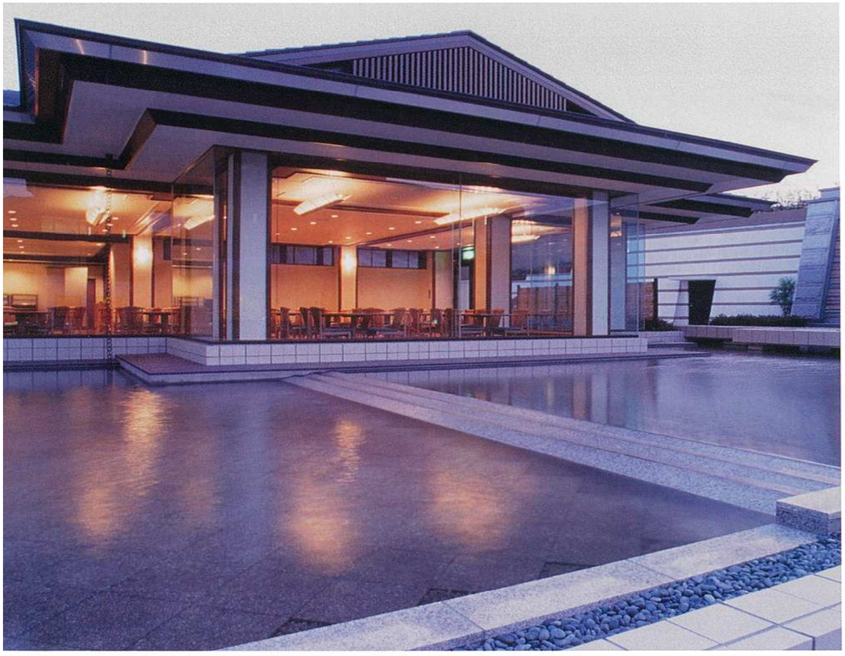
JR 逗子駅より約 20 分
20 minutes from JR Zushi Station

Breakfast, Lunch, Dinner

Cafeteria OAK オーク

営業時間

予約にて承ります（三日前まで）
朝食／7：30～9：00
昼食／11：30～13：30
夕食／18：00～19：30（L.O.）



朝食／ Breakfast

和洋食のブッフェスタイル
Buffet style with Japanese and Western breakfast

昼食／ Lunch

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夕食／ Dinner

主菜は二品から一品、副菜はお好みのものをご選択いただけるカフェテリア方式
Cafeteria style with main dish and a variety of side dishes

Banquet (August 20)

ホワイエ／ Foyer

Isospectral flows related to Frobenius-Stickelberger-Thiele polynomials

Xiangke Chang
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Motivated by an explicit formula studied in the modified Camassa-Holm (mCH) peakon lattice [2], we define a family of polynomials called the Frobenius-Stickelberger-Thiele (FST) polynomials previously introduced in [3]. For a specific choice of the deformation of the spectral measure, one is led to an integrable lattice (FST lattice), which is indeed an isospectral flow connected with a generalized eigenvalue problem. The spectral problem used previously in the study of the mCH peakon lattice is interpreted in terms of the FST polynomials together with the associated FST polynomials, resulting in a map from the mCH peakon lattice to a negative flow of the finite FST lattice. Furthermore, it is pointed out that the degenerate case of the finite FST lattice unexpectedly maps to the interlacing peakon ODE system associated with the two-component mCH equation studied in [1]. This talk is based on a joint work with Xingbiao Hu, Jacek Szmigielski and Alexei Zhedanov.

References

[1] X. Chang, X. Hu, and J. Szmigielski. Multipeakons of a two-component modified Camassa-Holm equation and the relation with the finite Kac-van Moerbeke lattice. *Adv. Math.*, **299** (2016), 1–35.

[2] X. Chang and J. Szmigielski. Lax integrability and the peakon problem for the modified Camassa-Holm equation. *Commun. Math. Phys.*, **358** (2018), 295–341.

[3] V. Spiridonov, S. Tsujimoto, and A. Zhedanov. Integrable discrete time chains for the Frobenius-Stickelberger-Thiele polynomials. *Commun. Math. Phys.*, **272** (2007), 139–165.

Bilinear equations and solutions of k-constrained DΔKP

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The k-constrained DΔKP is investigated from views of the spectral problem, bilinear equations and solutions. These bilinear equations can be reduced to those of the k-constrained KP, on the reverse direction the solution of the k-constrained KP can be used to constructed the one of the k-constrained DΔKP. As example, the double Wronskian solution of the semi-discrete AKNS hierarchy is derived from the one of the AKNS hierarchy.

Constructing two-dimensional optimal system of the group invariant solutions

Yong Chen
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To search for inequivalent group invariant solutions of two-dimensional optimal system, a direct and systematic approach is established, which is based on commutator relations, adjoint matrix, and the invariants. The details of computing all the invariants for two-dimensional algebra are presented, which is shown more complex than that of one-dimensional algebra. The optimality of two-dimensional optimal systems is shown clearly for each step of the algorithm, with no further proof. To leave the algorithm clear, each stage is illustrated with a couple of examples: the heat equation and the Novikov equation. Finally, two-dimensional optimal system of the (2+1) dimensional Navier-Stokes (NS) equation is found and used to generate intrinsically different reduced ordinary differential equations. Some interesting explicit solutions of the NS equation are provided.

Riemann-Hilbert Approach to Orthogonal Polynomials and Random Matrices

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In this talk, we first show the connections among the Riemann-Hilbert problem, orthogonal polynomials and random matrices. Then we show how to use Riemann-hilbert approach to analyze asympototic of orthogonal polynomials and random matrices.

References

[1] P Deift, Orthogonal polynomials and Random Matrices: A Riemann-Hilbert Approach, AMS, 2000

[2] P Deift, AR Its and X Zhou, A Riemann-Hilbert approach to asymptotic problems arising in the theory of random matrix models, and also in the theory of integrable statistical mechanics, *Annals Math.* **146** (1997), 149-235

[3] P Deift, T Kriecherbauer and KTR McLaughlin, Strong asymptotics of orthogonal polynomials with respect to exponential weights, *Commun. Pure Appl. Math.*, **52** (1999), 1491-1552

J. Phys. Soc. Japan **43** (1977), 1424–1433.

Convergence acceleration algorithms and discrete integrable systems

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The links between convergence acceleration algorithms and discrete integrable systems are presented. Some old and new results in this direction are reported.

Gauge transformations for the twisted derivation and quasideterminant solutions to the super KdV equation

Chun-Xia Li (A joint work with Jon Nimmo)
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Email: trisha_li2001@163.com

In this talk, I will first propose the concept of the twisted derivation and then present its gauge transformation which is expressed in terms of quasideterminants in general. We remark that the twisted derivation includes the normal derivative, the difference operator, the q-difference operator and super derivative as its special cases. Based on the gauge transformation, we are able to construct Darboux transformation and binary Darboux transformation for the super KdV equation, which can be regarded as a kind of noncommutative integrable systems due to the existence of fermion variables. By iteration, we finally arrive at quasideterminant solutions to the super KdV equation, which can be reduced to its existing superdeterminant solutions. We would like to mention that the above procedure could be used to derive quasideterminant solutions to the noncommutative KP equation, the $(2+1)$-dimensional non-Abelian Toda lattice equation, the non-Abelian Hirota-Miwa equation and the noncommutative $(2+1)$-dimensional q-discrete Toda lattice equation as well. This result reveals the great advantage of applying quasideterminants to deal with noncommutative integrable systems in a unified way.

References

[1] C.X. Li & J.J.C. Nimmo, Darboux transformations for a twisted derivation and quasideterminant solutions to the super KdV equation, *Proc. R. Soc. A.* **466** (2010), 2471–2493.

Multi-soliton solutions of the two-component Camassa-Holm system: Darboux transformation approach

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We present one more approach to construct multi-soliton solutions of an integrable two-component Camassa-Holm (CH2) system. With help of a reciprocal transformation and a gauge transformation, we relate the CH2 system to the negative flow of the Broer-Kaup or two-boson hierarchy. The solutions of this negative flow are given in terms of Wronskians via Darboux transformation. Then the multi-soliton solutions of the CH2 system are recovered in parametric form by inverting the reciprocal transformation and the gauge transformation. This is a joint work with Gaihua Wang and Nianhua Li.

Orbital Stability of Peaked Solitary Waves with arbitrary order of Nonlinearities

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Email: liuxiaochuan@mail.xjtu.edu.cn

The stability of solitary waves (or solitons) is one of the fundamental qualitative properties of the solutions of nonlinear waves equations. In this talk, I will give a review of what have been obtained about the orbital stability of the peaked solitary waves (or peakons), present some recent results for the higher-order Camassa-Holm-type dispersive equations and investigate the effect of higher-order nonlinearity on the structure of stability.

Multi-place systems and multi-place physics

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In this report, some new physics and new mathematics are summarised for nonlocal multi-place systems. Multi-place nonlocal systems can be systematically derived from multi-component local equations via discrete symmetry groups constructed by parity, time reversal, exchanges and charge conjugate[1]. Two and four place nonlocal NLS, KP and Boussinesq systems are especially discussed. Multiple soliton solutions of two and four place nonlocal systems are explicitly obtained via symmetry-antisymmetry separation method related to the discrete symmetry operators. From exact soliton solutions, one can find that the multi-place nonlocal systems are related to the multi-place correlations, entanglements, interactions[2] and the nonlinear excitations with special discrete symmetries of the initial or boundary conditions. Nonlocality may lead to some classical prohibitions [3], transitions [4] structure modifications of the basic nonlinear excitations.

References

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Abundant exact solutions to the discrete complex mKdV equation by Darboux transformation

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Abstract:

An N-fold Darboux transformation is constructed for the discrete complex modified Korteweg-de Vries equation in term of determinants. Through the obtained one-fold and two-fold Darboux transformation, a variety of new exact solutions, including an anti-dark soliton solution, a breather solution, a periodic solution, and a two-soliton solution, are derived from a nonzero constant and plane-wave seed solution. Via numerical simulation, a new kind of dynamical behavior of the two-soliton solution is exhibited, which tells that the two-soliton solution includes an anti-dark solitary wave and a W-shaped solitary wave.

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Completion of the Ablowitz-Kaup-Newell-Segur integrable coupling

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Integrable couplings are associated with non-semisimple Lie algebras. We propose a new method to generate new integrable systems through making perturbation in matrix spectral problems for integrable couplings, which is called the 'completion process of integrable couplings'. As an example, the idea of construction is applied to the Ablowitz-Kaup-Newell-Segur integrable coupling. Each equation in the resulting hierarchy has a bi-Hamiltonian structure furnished by the component-trace identity [1].

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A numerical study of the 3-periodic wave solutions to KdV-type equations

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In this talk, by using the direct method of calculating periodic wave solutions proposed by Akira Nakamura, we present a numerical process to calculate the 3-periodic wave solutions to several KdV-type equations: the Korteweg-de Vries equation, the Sawada-Koterra equation, the Boussinesq equation, the Ito equation, the Hietarinta equation and the (2+1)-dimensional Kadomtsev-Petviashvili equation. Some detailed numerical examples are given to show the existence of the three-periodic wave solutions numerically.

A general nonlocal nonlinear Schrödinger equation with shifted parity, charge-conjugate and delayed time reversal

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A general nonlocal nonlinear Schrödinger equation with shifted parity, charge-conjugate and delayed time reversal is derived from the nonlinear inviscid dissipative and equivalent barotropic vorticity equation in a β-plane. The modulational instability (MI) of the obtained system is studied, which reveals a number of possibilities for the MI regions due to the generalized dispersion relation that relates the frequency and wavenumber of the modulating perturbations. Exact periodic solutions in terms of Jacobi elliptic functions are obtained, which, in the limit of the modulus approaches unity, reduce to soliton, kink solutions and their linear superpositions. Representative profiles of different nonlinear wave excitations are displayed graphically. These solutions can be used to model different blocking events in climate disasters. As an illustration, a special approximate solution is given to describe a kind of two correlated dipole blocking events. The work has been published in Ref. [1].

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Virasoro symmetries for Drinfeld-Sokolov hierarchies and equations of Painlevé type

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We derive a series of Virasoro symmetries for the (generalized) Drinfeld-Sokolov hierarchy associated to an arbitrary affine Kac-Moody algebra with two certain gradations. By imposing the Virasoro constraints to the hierarchy, we obtain its solutions of Witten-Kontsevich and of Brezin-Gross-Witten types, and solutions characterized by certain ordinary differential equations of Painlevé type. We also show the existence of affine Weyl group actions on solutions of such Painlevé type equations, which agrees with the theory of Noumi and Yamada on affine Weyl group symmetries of the Painlevé type equations. This is a collaboration with Si-Qi Liu and Youjin Zhang.

Darboux transforms for KdV-type hierarchies associated to affine Kac-Moody algebra of B-type

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Drinfeld-Sokolov in [1] associated to each affine Kac-Moody algebra $\mathcal{G}$ a $\mathcal{G}$-hierarchy of soliton equations and constructed a $\mathcal{G}$-KdV hierarchy on a cross section of certain gauge action by pushing down the $\mathcal{G}$-hierarchy along the gauge orbits to the cross section. For example, the $\hat{B}_1^{(1)}$-KdV hierarchy is the standard KdV hierarchy with the KdV

$$q_t = q_{xxx} - 3qq_x \tag{1}$$

In this talk, we will discuss KdV-type flows associated to affine Kac-Moody algebra of B-type and using loop group factorization in [2] to construct Darboux transforms for these flows.

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Riemann-Hilbert approach for the integrable modified Camassa-Holm equation with cubic nonlinearity

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We investigate the initial value problem for the modified Camassa-Holm (m-CH) equation with cubic nonlinearity. The mCH equation is known to be integrable, which we mean it admits an Lax pair. We formulate the initial value problem as an associate vector Riemann-Hilbert problem, which allows us to give a parametric representation of the solution to the initial value problem in terms of the solution of the Riemann-Hilbert problem.

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On quasi-triviality of a class of PDEs

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Explicit expression for quasi-triviality of scalar non-linear PDE is under consideration.

On a nonlocal Alice-Bob system of nonlinear Schrödinger equation: Bilinear Bäcklund transformation, Darboux transformation and soliton solutions

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Motivated by the increasing desire to understand many events in natural and social sciences that happened at different space-times and usually entangled or correlated closely and can be described by using Alice and Bob, Lou put forward several physical and mathematical meaningful new Alice-Bob systems quite recently. Here, a new such kind of nonlinear Schrödinger equation (AB-NLS) derived from the well-known AKNS system is investigated. We construct not only a bilinear Bäcklund transformation for the unreduced AB-NLS by the Hirota method, but also an n-fold Darboux transformation for the reduced AB-NLS. Armed with them we obtain different kinds of solutions for the new AB-NLS, which are quite different from that of the NLS equation.

Invariant subspaces of biconfluent Heun operators and special solutions of Painlevé IV

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In this talk, we show that there is a full correspondence between the parameters space of the degenerate biconfluent Heun connection (BHC) and that of Painlev'e IV that admits special solutions. The BHC degenerates when either the Stokes' data for the irregular singularity at degenerates or the regular singular point at the origin becomes an apparent singularity. We show that if the BHC is written as isomonodromy family of biconfluent Heun equations (BHE), then the BHE degenerates precisely when it admits eigen-solutions of the biconfluent Heun operators, after choosing appropriate accessory parameter, of specially constructed invariant subspaces of finite dimensional solution spaces spanned by parabolic cylinder functions. We have found all eigen-solutions over this parameter space apart from three exceptional cases after choosing the right accessory parameters. These eigen-solutions are expressed as certain finite sum of parabolic cylinder functions. We extend the above sum to new convergent series expansion in terms of parabolic cylinder functions to the BHE. The infinite sum solutions of the BHE terminates precisely when the parameters of the BHE assumes the same values as those of the degenerate biconfluent Heun connection except at three instances after choosing the right accessory parameter [1].

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Integrable mappings arising from open boundary reductions

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In the study of Cauchy problems for two-dimensional integrable quad-equations on $\mathbb{Z}^2$ lattices, integrable mappings can be obtained through the so-called periodic reductions. In this talk, based on a recent study of initial-boundary-value problems for integrable quad-equations by Caudrelier, Crampé & **CZ** (SIGMA 10(2014):014), another type of reductions, the *open boundary reductions*, is provided. Instead of imposing periodicity on the underlying lattice, initial-boundary-value problems are posed on a strip with two parallel boundaries. The integrability is relying on the so-called boundary consistency, which is the defining integrable criterion for quad-equations with boundary. Naturally, integrable initial-boundary problems give rise to classes of integrable mappings.This work is in collaboration with Peter van der Kamp and Vincent Caudrelier.

On multidimensional consistency

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The property of multidimensional consistency plays an important role in the investigation of lattice equations. In this talk first I will briefly recall this property and some examples of multidimensionally consistent equations such as discrete Burgers, lattice potential KdV, Miwa's equation, and so on. Then I will introduce some extensions based on multidimensionally consistent equations, such as two-component version and edge systems. (In collaboration with Peter van der Kamp and Danda Zhang.)

Permutation matrices and multi-component coupled integrable lattice systems

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In this talk we would like to introduce a method for generating vector-value integrable analogies of integrable lattice systems or integrable differential-difference equations. The basic ingredient of the method is to insert permutation matrices. We formulate the zero-curvature representations and Hamiltonian structures of the resulting vector lattice systems.We take Toda lattice as illustrative example.

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On A Reverse Space-Time Nonlocal Sasa-Satsuma Equation

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The Sasa-Satsuma equation is an integrable high-order nonlinear Schrödinger equation, and also can be viewed as a complex modified KdV-type equation. It can describe the propagation of femtosecond pulses in optical fibers. In this talk, inspired by the work on nonlocal integrable systems of Ablowitz and Mussliman, we introduce a reverse space-time nonlocal Sasa-Satsuma equation, and derive its solutions with the Darboux transformation method. Periodic solutions, and some localized solutions, such as dark soliton, W-shaped soliton, M-shaped soliton and breather soliton of the reverse space-time nonlocal Sasa-Satsuma equation are constructed. This is a joint work with C. Q. Song, and D. M. Xiao.

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Extended affine Weyl groups of BCD-type: their Frobenius manifolds and Landau–Ginzburg superpotentials

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This is a joint work with B.Dubrovin, Ian Strachan and Y-J. Zhang [2]. For the root systems of type B_l, C_l and D_l, we generalize the result of [1] by showing the existence of Frobenius manifold structures on the orbit spaces of the extended affine Weyl groups that correspond to any vertex of the Dynkin diagram instead of a particular choice made in [1]. It also depends on certain additional data. We also construct Landau–Ginzburg superpotentials for these Frobenius manifold structures.

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Construction of Laurent-Jacobi matrices with prescribed eigenvalues via orthogonal polynomials

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Solving inverse eigenvalue problems (IEPs) is an important subject in numerical linear algebra. Chu and Golub gave a comprehensive review for IEPs [1]. The problem to construct a matrix having prescribed eigenvalues is one of IEPs and called the structured IEP. It is known that orthogonal polynomials are closely related to matrix eigenproblems. For example, in [2], Hendriksen and Nijhuis associated orthogonal Laurent polynomials with the pentadiagonal matrices called Laurent-Jacobi matrices.

In this talk, we present an algorithm to construct Laurent-Jacobi matrices with prescribed eiganvalues by making use of a relation between orthogonal Laurent polynomials and Laurent biorthogonal polynomials [3].

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Common property of asymptotic behavior of some probabilistic cellular automata

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Probabilistic Burgers cellular automaton (PBCA) is equivalent to parallel updated totally asymmetric simple exclusion process (TASEP). In this presentation, utilizing transition matrices and primitive combinatorial methods, we propose some conjectures for asymptotic distribution of PBCA and derive fundamental diagram (FD) which shows relations between density and mean flow of particles by conjectures. In order to have the conjectures, we utilize a property that each element of an eigenvector for eigenvalue 1 of transition matrices of PBCA for some parameters can be factorized. Moreover, we propose some extended systems of PBCA which have the common property like PBCA.

Initial value problem of max equation with an external variable and of a system of max-equations

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We can propose the non-autonomous equation by introducing the external parameters into the autonomous equation. In this presentation, we will show some examples of non-autonomization of max equation preserving the polynomial order of complexity of solutions. There are some autonomous max equations with this order, but if we introduce the non-autonomous external parameter, the complexity of solution often grows and changes to exponential order. To preserve the polynomial order, we need to specify the form of max equation with an external parameter. Moreover, we will show some examples of system of max equations on two dependent variables with polynomial order of complexity. In this case, one variable can be considered to play a role of external term to the other.

Ultradiscretization with parity variables of the hard-spring equation and its conserved quantity

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Ultradiscretization, proposed by T. Tokihiro, D. Takahashi, J. Matsukidaira and J. Satsuma in 1996, is a limiting procedure transforming a given subtraction-free difference equation into the piecewise-linear equation, which can be regarded as a time evolution rule of a cellular automaton (CA). This procedure may enable us to export knowledge from the analysis of differential equations to CA and develop research on nonlinear science.

However, "subtraction-free" is a strict restriction in the traditional ultradiscretization. An approach to overcome this restriction is "ultradiscretization with parity variables" (p-ultradiscretization) [1]. In this procedure, we consider the sign of dependent variables in each time evolution step and obtain a set of conditional piecewise-linear equations. Do essential qualitative properties of the original equation survive through p-ultradiscretization?

A difference equation

$$\frac{x_{n+1} - 2x_n + x_{n-1}}{2\delta^2} + c_1(x_{n+1} + x_{n-1}) + 2c_2 x_n + c_3 {x_n}^2 (x_{n+1} + x_{n-1}) = 0 \quad (1)$$

is a discrete analogue of the hard-spring equation $x''(t) + ax(t) + b(x(t))^3 = 0$ and admits one conserved quantity [2]. This integrable discrete system may be a suitable target for studying fundamental property of p-ultradiscretization.

In this talk, we report our research [3] on p-ultradiscrete analogue of (1) and its conserved quantity. We study three objects:

(i) solutions of initial value problems for the ultradiscrete equation,

(ii) behaviour of p-ultradiscretized conserved quantity,

(iii) approximated solutions of (1) by the leading term.

In each object, the results are classified into some types. Moreover, within our experiments, we observe correspondence of these types among objects.

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Algebro-geometric solutions of lattice soliton equations arising from Toda lattice hierarchy

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Lattice soliton equations, such as lattice potential KdV equation [1]

$$(x_{m+1,n} - x_{m,n+1})(x_{m,n} - x_{m+1,n+1}) = p^2 - q^2,$$

lattice Schwarzian KdV equation [1]

$$p^2 (x_{m,n} - x_{m+1,n})(x_{m,n+1} - x_{m+1,n+1}) = q^2 (x_{m,n} - x_{m,n+1})(x_{m+1,n} - x_{m+1,n+1}),$$

lattice Boussinesq system [3]

$$y_{m+1,n} = x_{m,n}x_{m+1,n} - z_{m,n}, \quad y_{m,n+1} = x_{m,n}x_{m,n+1} - z_{m,n},$$
$$z_{m+1,n+1} = x_{m,n}x_{m+1,n+1} - y_{m,n} + \frac{p-q}{x_{m+1,n} - x_{m,n+1}},$$

lattice Schwarzian Boussinesq equation [2]

$$\begin{aligned}
&(x_{m+2,n+2} - x_{m+1,n+2})(x_{m,n+2} - x_{m+1,n+1})(x_{m,n+1} - x_{m,n}) \\
&\times \Big\{ q^3 (x_{m+2,n+1} - x_{m+2,n})(x_{m+1,n+1} - x_{m+1,n}) \\
&\qquad - p^3 (x_{m+2,n+1} - x_{m+1,n+1})(x_{m+2,n} - x_{m+1,n}) \Big\} \\
&= (x_{m+2,n+2} - x_{m+2,n+1})(x_{m+2,n} - x_{m+1,n+1})(x_{m+1,n} - x_{m,n}) \\
&\quad \times \Big\{ p^3 (x_{m+1,n+2} - x_{m,n+2})(x_{m+1,n+1} - x_{m,n+1}) \\
&\qquad - q^3 (x_{m+1,n+2} - x_{m+1,n+1})(x_{m,n+2} - x_{m,n+1}) \Big\}
\end{aligned}$$

are investigated from the viewpoint of Toda lattice hierarchy [4]. We will show that these equations can be obtained as reductions of the Toda lattice hierarchy, and discuss their solutions (soliton-type solutions, algebro-geometric solutions) in unified manner.

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The discrete Toda equation proves the Aztec diamond theorem

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Tilings of the Aztec diamonds are combinatorial objects which we can exactly enumerate; the number of (domino-)tilings of the Aztec diamond of order n is equal to $2^{\frac{n(n+1)}{2}}$. Many different proofs are given to this fact including combinatorial ones and algebraic ones. (See [1, 2] for example.) In this talk a new proof by using the discrete two-dimensional (2D) Toda equation will be shown. The proof is based on the observation that we can construct a generating (or partition) function for tilings of the Aztec diamonds and its product expression from any solution to the discrete 2D Toda equation.

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Wakimoto realization of the quantum affine superalgebra $U_q(\widehat{sl}(M|N))$

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A bosonization of the quantum affine superalgebra $U_q(\widehat{sl}(M|N))$ is presented for an arbitrary level $k \in \mathbf{C}$. We call this bosonization Wakimoto realization [1]. This talk is based on [2].

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Exceptional $q = -1$ polynomials

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Exceptional orthogonal polynomials are important extensions of the classical families including but not limited to the Hermite, Laguerre and Jacobi polynomials. An Bochner type theorem has been presented to address the fact that every system of exceptional orthogonal polynomials can be obtained from a classical orthogonal polynomial system through a sequence of Darboux transformations [1]. Note that in this context the classical orthogonal polynomials are limited to the Hermite, Laguerre and Jacobi polynomials, sometimes also be referred to as the "very" classical orthogonal polynomials. In general, polynomials in the Askey-Wilson scheme all can be called classical. The term classical means that apart from the three-term recurrence relations, these polynomials satisfy also an eigenvalue equation. Recently, several new families of polynomial systems which appear by taking a nontrivial limit $q = -1$ on orthogonal polynomials from the Askey-Wilson scheme have been identified classical. They satisfy eigenvalue problems with differential/difference operators of Dunkl type. Specifically, these polynomials are the Bannai-Ito polynomials, the big -1-Jacobi and the little -1-Jacobi polynomials [3, 4, 5]. Unlike the previous cases, the associated Dunkl-type operators are of first-order which cannot be factorized into two first-order as it was performed in an ordinary Darboux transformation. We apply a generalized Darboux transformation by making use of a pair of intertwining relations satisfied by the Dunkl-type operators. In this way we derive some exceptional extensions of these $q = -1$ polynomials [2]. An interesting fact of these exceptional orthogonal polynomial systems is that in several cases the corresponding degree sequences are consist of even numbers only, for example, $0,2,2,4,4,\cdots$. We further study their ladder operators and the associated algebraic relations to address this fact.

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Generalized box–ball systems and nonautonomous discrete Toda lattices

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We first give a connection between the generalized box–ball system proposed by Hatayama *et al.* [1] and the nonautonomous discrete hungry Toda lattice [2]. Studies on the connection also derived another deautonomization of the discrete hungry Toda lattice and a novel generalized box–ball system [3]. We discuss the novel generalized box–ball system, e.g., its KP type time evolution equation, particular solutions, and so on.

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Spatial pattern of ultradiscretizable discrete Gray-Scott model and Turing instabilities of its equilibrium solutions

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Gray-Scott model:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - uv^2 + a(1-u), \\ \dfrac{\partial v}{\partial t} = D\dfrac{\partial^2 v}{\partial x^2} + uv^2 - bv, \end{cases}$$

where $u := u(t,x),\ v := v(t,x),\ t \geq 0,\ x \in \Omega_c \subset \mathbb{R},\ a, b, D > 0$, is a reaction-diffusion system and whose solutions give various spatial patterns[1, 3]. In this talk, we propose a discretization and an ultradiscretization of Gray-Scott model [2]. Ultradiscretization is a limiting procedure transforming a given difference equation into a cellular automaton. The ultradiscrete system is directly related to the elementary cellular automaton Rule 90 which gives a Sierpinski gasket pattern. We also discuss relation between spatial patterns of the discrete system and Turing instability.

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Quantum walks on graphs associated with bivariate Krawtchouk polynomials

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Orthogonal polynomials play a central role in analysing 1-dimensional *XX* spin lattice model, which is equivalent to continuous time quantum walks. Especially, the model associated with Krawtchouk polynomials exhibits perfect state transfer and relates to quantum walks on hypercube.
In this talk, we shall focus on bivariate extension of Krawtchouk polynomials and consider quantum walks on the associated graphs. The corresponding 2-dimensional *XX* spin lattice model is shown to be derived from this quantum walk and its analysis is given by using the properties of these polynomials. This talk is based on joint work with Satoshi Tsujimoto and Luc Vinet [1].

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The best constant of discrete Sobolev inequalities

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The best constant of a certain class of Sobolev inequalities was found by Talenti [1] by through variational methods. In our works, we found the best constant of a different class of Sobolev inequalities through a different approach, which is based on the theory of Green function and reproducing kernel [2].

In this talk, starting from boundary value problems of difference equations, we derive discrete Sobolev inequalities and its best constant. In particular, discrete Sobolev inequalities on graphs, including periodic lattice and C60 buckeyball, are investigated in a detailed manner [3, 4].

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Discrete and Ultradiscrete mixed Soliton Solutions

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We propose a new type of soliton equation. It is obtained from the generalized discrete BKP equation and admits different types of soliton solutions[1]. We also propose ultradiscrete analogues of them. The ultradiscrete equation also admits the mixed solution, which includes the original Box-Ball system in a special case.

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Determinant formulas for tau functions of q-Painlevé systems in terms of q-Nekrasov partition functions

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In [1], the authors gave an explicit formula for general solutions to the q-difference Painlevé VI equation, in terms of q-Nekrasov partition functions. The basic idea is to construct fundamental solutions of the Lax pair for q-P_{VI} by using the 5-point q-Nekrasov partition function of rank 2 with the degenerate condition.

Let us introduce the q-Nekrasov partition function. For partitions λ, μ, we set

$$N_{\lambda,\mu}(u) = \prod_{\square\in\lambda}\left(1 - q^{-\ell_\lambda(\square)-a_\mu(\square)-1}u\right)\prod_{\square\in\mu}\left(1 - q^{\ell_\mu(\square)+a_\lambda(\square)+1}u\right),$$

which we call a Nekrasov factor. The q-Nekrasov partition function of rank 2 reads as

$$Z\left(\begin{matrix} & \theta_m & \theta_{m-1} & \cdots & \theta_1 & \\ \theta_{m+1} & \sigma_{m-1} & \sigma_{m-2} & \cdots & \sigma_1 & \theta_0\end{matrix}; x_m, \dots, x_1\right)$$
$$= \sum_{\boldsymbol{\lambda}^{(1)},\dots,\boldsymbol{\lambda}^{(m-1)}} \prod_{p=1}^{m-1} x_p^{|\boldsymbol{\lambda}^{(p)}|} \cdot \frac{\prod_{p=1}^{m}\prod_{\epsilon,\epsilon'=\pm} N_{\lambda_\epsilon^{(p)},\lambda_{\epsilon'}^{(p-1)}}(q^{\epsilon\sigma_p-\theta_p-\epsilon'\sigma_{p-1}})}{\prod_{p=1}^{m-1}\prod_{\epsilon,\epsilon'=\pm} N_{\lambda_\epsilon^{(p)},\lambda_{\epsilon'}^{(p)}}(q^{\epsilon\sigma_p-\epsilon'\sigma_p})}.$$

We have set $\sigma_0 = \theta_0$, $\sigma_m = \theta_{m+1}$, $\boldsymbol{\lambda}^{(0)} = \boldsymbol{\lambda}^{(m)} = (\emptyset,\emptyset)$, and the sum is taken over all pair of partitions $\boldsymbol{\lambda}^{(p)} = (\lambda_+^{(p)}, \lambda_-^{(p)})$, $p = 1, \dots, m-1$. If $m = 2$, $\theta_1 = 1/2$, and $\sigma_1 = \theta_0 \pm 1/2$, then the above function becomes the q-hypergeometric series ${}_2\varphi_1(x_1/x_2)$. We call the condition $\theta_i = 1/2$, and $\sigma_i = \sigma_{i-1} \pm 1/2$ the degenerate condition.

In my talk, I will discuss the rank N case. I explain how to construct a fundamental solution of a connection preserving deformation of rank N in terms of q-Nekrasov partition function of rank N. As a result, I derive determinant formulas for tau functions of q-Painlevé systems in terms of q-Nekrasov partition functions. When $N = 2$, they are bilinear equations for tau functions.

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Consistency around a cuboctahedron

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In the theory of discrete integrable systems, the classification of lattice equations using the consistency around a cube (CAC) property [1, 2, 4, 5, 6] are well known. The resulting equations are collectively called ABS equations.

In this talk, we show new classification of lattice equations using a consistency around a cuboctahedron (CACO) property. This work is based on the collaboration with Prof. Nalini Joshi (The University of Sydney) [7].

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A fuzzification scheme for three states cellular automata

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A cellular automaton (CA) is an array of cells that evolves through discrete time steps according to the local rule. A familiar one is an elementary cellular automaton (ECA), which is the one-dimensional CA with two states, say 0 and 1, and radius one. Among 256 rules, ECA Rule 184 is also known as the simplest modeling of traffic flow. To investigate the dynamics of ECA, Cattaneo et al. [1] proposed the continuous counterpart of ECA. Their approach is the "fuzzification" of the local rules expressed in the disjunctive normal form, and the yielding one is called fuzzy CA. Theoretical results on fuzzy CA, such as the conservation law, convergence and self-oscillation are given by Betel and Flocchini (e.g. [2, 3]). Since two states $\{0, 1\}$ are extended to the interval $[0, 1]$, fuzzy CA is often visualized in gray-scaled space-time diagrams.

In this talk, we develop the fuzzification scheme for three states CA. Slow to Start Rule for traffic flow model [4] is an example of three states CA, where three states stand for 'cars enabled to move', 'cars forced to stop' and 'absence of cars'. In our fuzzification process, we assume that three states are not a sequence of numbers, like 0, 1 and 2, but the vertices of a triangle. Then we get the fuzzy CA in which the states of cells are defined inside the triangle. Regarding the inner point of the triangle as the mixture of three states, we can visualize a fuzzy CA in RGB colors. Further, we present some properties of our fuzzy CA, such as the conservation law and the convergence.

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C program to solve the initial value problem of max-min equations

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Complexity of formal solutions to max-min evolutional equation grows exponentially as time develops. There are useful formulas on max (min) operations to reduce this complexity. If we define the class of equations by the minimum complexity obtained by the reduction, they can be classified into the polynomial or exponential order. In order to identify the class, we can make the computer program to arrange and reduce the expression of solutions. Symbolic manipulation software like Mathematica is useful and strong tool to realize this, but the calculation cost becomes often huge. Therefore, we develop the C program which can operate max-min symbolic manipulation using pointer and structure of C language. We will explain about the structure of our C program and results obtained by applying the concrete max-min equations.

A general prolongation formulation for symmetries of differential-difference equations

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Symmetry methods have played fundamentally important roles in analysing the integrability and solvability of differential equations since S. Lie's pioneering work in 1880s. During last decades, a lot of efforts were contributed to extend these methods to difference equations and differential-difference equations (DDEs), both of which not only come from discretisations of differential equations but also are dynamical models themselves.

Recently, people realised that symmetry techniques of DDEs, somehow 'singular' between continuous and fully discrete cases, do not simply mimic those of differential/difference equations; for instance, the authors tried to fix previous work via a semi-discrete continuum limit of the difference case in the paper [2] published in 2010. In this talk, I will introduce a rigorous analysis about symmetries of DDEs based on [3]; see also [4] for its comparison with previous literature, e.g. [1, 2]. Two main questions about symmetries of DDEs will be answered: 1) *Why and how they are different from the differential and difference cases?* 2) *What is the symmetry prolongation formulation and how to calculate symmetries using the linearized symmetry condition?*

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Cluster algebra and q-Painlevé equations

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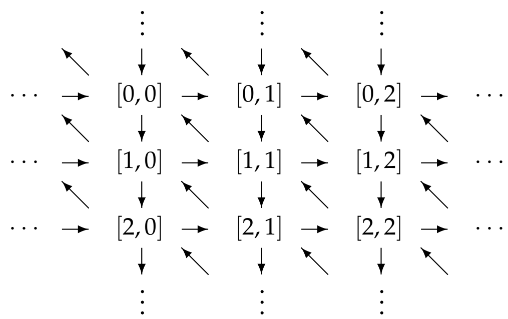
The cluster algebra was introduced by Fomin and Zelevinsky. It is a variety of commutative ring described in terms of cluster variables and coefficients. Its generating set is defined by an operation called a mutation which transforms a seed consisting of cluster variables, coefficients and a quiver. Then new cluster variables (reps. coefficients) are rational in original cluster variables and coefficients (reps. coefficients). Hence we can obtain various discrete integrable systems from mutation-periodic quivers as relations satisfied by cluster variables and coefficients.



In this talk, we consider the above quiver. We always assume that

$$[i,j] = [i+mn,j] = [i,j+m] \quad (m,n \in \mathbb{N},\ m>1,\ mn>2).$$

Hence the quiver can be regarded as the one on a torus. This quiver is invariant under some compositions of mutations and permutations of vertices of quivers. Those operations generate a group of birational transformations which is isomorphic to an extended affine Weyl group of type $(A_{mn-1}+A_{m-1}+A_{m-1})^{(1)}$. And translations of this group provides higher order generalizations of the q-Painlevé equations. In this talk we explain its detail. If time permits, we discuss q-hypergeometric solutions and degeneration structures of the q-Painlevé systems.

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Volterra-type hierarchies for specialized hypergeometric tau functions

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1 Hypergeometric tau functions

The hypergeometric tau functions[1] are tau functions of the 2D Toda hierarchy of the form

$$\mathcal{T}(s,\boldsymbol{t}) = \sum_{\lambda\in\mathcal{P}} S_\lambda(\boldsymbol{t}) g_\lambda(s) S_\lambda(-\bar{\boldsymbol{t}}),$$

where $\boldsymbol{t} = (t_k)_{k=1}^\infty$ and $\bar{\boldsymbol{t}} = (\bar{t}_k)_{k=1}^\infty$ are two sets of the time variables, and $\mathcal{P}$ is the set of all partitions $\lambda = (\lambda_i)_{i=1}^\infty$. $S_\lambda(\boldsymbol{t})$'s are the Schur functions

$$S_\lambda(\boldsymbol{t}) = \det(S_{\lambda_i-i+j}(\boldsymbol{t}))_{i,j=1}^\infty, \quad \sum_{n=0}^\infty S_n(\boldsymbol{t}) z^n = \exp\left(\sum_{k=1}^\infty t_k z^k\right).$$

The coefficients $g_\lambda(s)$ are defined by the somewhat formal expression

$$g_\lambda(s) = \frac{\prod_{i=1}^\infty g_{\lambda_i-i+s+1}}{\prod_{i=1}^\infty g_{-i+1}}$$

for a diagonal matrix $g = \mathrm{diag}(g_i)_{i=-\infty}^\infty \in \mathrm{GL}(\infty)$, representing the matrix elements of an associated Clifford operator in the fermionic Fock space.

2 Specialized hypergeometric tau functions

We consider the specialized tau function

$$\mathcal{T}(s,\boldsymbol{t}) = \mathcal{T}(s,\boldsymbol{t},-\boldsymbol{c}),$$

where $\boldsymbol{c} = (c_k)_{k=1}^\infty$ are a set of constants, and g is specialized to

$$g = e^{\beta(\Delta-1/2)^2/2} Q^\Delta, \quad \Delta = \mathrm{diag}(i)_{i=-\infty}^\infty$$

where β and Q are parameters. The fermionic matrix elements $g_\lambda(s)$ can be expressed as

$$g_\lambda(s) = \exp\left(\frac{\beta}{2}\left(\kappa(\lambda) + 2s|\lambda| + \frac{4s^3 - s}{12}\right)\right) Q^{|\lambda|+s(s+1)/2},$$

where $|\lambda| = \sum_{i=1}^\infty \lambda_i$ and $\kappa(\lambda) = \sum_{i=1}^\infty \lambda_i(\lambda_i - 2i + 1)$. The 2D Toda tau function $\mathcal{T}(s,\boldsymbol{t},\bar{\boldsymbol{t}})$ with these coefficients is known to be a generating function of the double Hurwitz numbers of $\mathbb{CP}^1$ [2].

We are particularly interested in the case where $\bar{\boldsymbol{t}} = -\boldsymbol{c}$ is further specialized to the following values:

$$\text{(i)} \quad c_k = \delta_{k,1} \qquad \text{(ii)} \quad c_k = \frac{q^{k/2}}{k(1-q^k)} \tag{1}$$

As we show, these two cases are related to *integrable hierarchies of the Volterra type.* These systems emerge as a reduction (or a subsystem) of *the lattice KP hierarchy.*

3 Volterra-type hierarchies in lattice KP hierarchy

The specialized hypergeometric tau function $\mathcal{T}(s, \boldsymbol{t})$ yields a solution of *the lattice KP hierarchy*

$$\frac{\partial L}{\partial t_k} = [B_k, L], \quad L = \Lambda + u_1 + u_2\Lambda^{-1} + \ldots, \quad \Lambda = e^{\partial_s}.$$

The Lax operator L can be expressed as $L = W\Lambda W^{-1}$ in terms of the dressing operator $W = 1 + w_1\Lambda^{-1} + w_2\Lambda^{-2} + \ldots$. One can thereby define $\log L$ (logarithm) and L^α (fractional or irrational power) as

$$\log L = W \cdot \log\Lambda \cdot W^{-1}, \quad L^\alpha = W\Lambda^\alpha W^{-1}.$$

Theorem 1 *The logarithm of L for the case (i) of (1) can be expressed as*

$$\log L = \log\Lambda + u\Lambda^{-1} = \partial_s + ue^{-\partial_s}, \quad u = u(s, \boldsymbol{t}). \tag{2}$$

Theorem 2 *Let β be parametrized as $\beta = (\tau + 1)\log q$ in the case (ii) of (1). Then the fractional power $L^{1/(\tau+1)}$ of L for this case can be expressed as*

$$L^{1/(\tau+1)} = (1 + u\Lambda^{-1})\Lambda^{1/(\tau+1)}, \quad u = u(s, \boldsymbol{t}). \tag{3}$$

If τ is equal to a positive integer N, (3) coincides with the Lax operator of the $N+1$-step Bogoyavlensky-Itoh (*aka* hungry Lotka-Volterra) hierarchy [3, 4] on the fractional lattice $\mathbb{Z}/(N+1)$. (3) is its large-N (or continuum) limit. These Volterra-type hierarchies thus capture the essential integrable structure of the specialized hypergeometric tau functions.

Let us also mention that the case (i) is related to the single Hurwitz numbers of $\mathbb{CP}^1$, for which Theorem 1 is reported in a recent paper [5]. The case (ii) is related to the cubic Hodge integrals [6].

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Soliton solutions of the DKP equation and networks

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Soliton solutions of the KP equation have Wronskian representation and their interactions are closely related to permutations and networks. By contrast, soliton solutions of the DKP (coupled KP) equation have Wronski-type Pfaffian representation and their interactions still have not been clarified clearly. In this presentation, we explain the relation between networks and soliton solutions of the DKP equation using normalization of skew-symmetric matrices and their networks. This is joint work with Shinya Kido, Yasuyuki Watanabe, Ken-ichi Maruno and Saburo Kakei.

On classical orthogonal polynomials from Jacobi to -1 polynomials

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We consider several types of eigenvalue problems related to the classical orthogonal polynomials such as the Jacobi polynomials, the Krawtchouk polynomials, the Askey-Wilson polynomials and so on. The first half of this talk will be devoted to the algebraic approaches for deriving the Askey-Wilson polynomials. We will show that q-oscillator algebra can acheive the Askey-Wilson algebra and that the double affine Hecke algebra of rank 1 naturally leads to circle analogs of the Askey-Wilson polynomials. In the second half of this talk we consider the eigenvalue problem associated with the Dunkl-type differential operator from which we can construct several classes of "-1" polynomial systems. The results of this talk are based on the joint work with L. Luo, L. Vinet and A. Zhedanov.

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On the direct and inverse scattering problems for udKdV

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We solve the direct scattering problem for the ultradiscrete Korteweg de Vries (udKdV) equation over the real numbers. More precisely, we construct explicit bound state and non-bound state eigenfunctions, for any initial potential with compact support for the scattering problem for udKdV. We then go on to show how to reconstruct the potential in the scattering problem at any time, using an ultradiscrete analogue of a Darboux dressing transformation, based on data uniquely characterising the soliton content and a 'background'. These data are obtained from the initial potential by Darboux undressing transformations. This is joint work with J.J.C. Nimmo and C. Gilson [1].

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Construction of exact soliton solutions in the spinor $F = 1$ Bose-Einstein condensates by Hirota bilinear method

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Bose-Einstein condensates (BECs) have received extensive attention since their first experimental realization in 1995. Multi-component BECs can be created by overlapping two single-component BECs with atoms in two spin (hyperfine) states or mixtures of two different atomic species by using far-off-resonant optical techniques for trapping of ultracold atomic gases. This gave rise to the observation of various phenomena that are not present in single-component BECs. A spinor condensate formed by atoms with spin F is described by a macroscopic wave function with $2F + 1$ components.

The integrable three components Gross-Pitaevskii (GP) equations for $F = 1$ spinor BECs in one-dimension without external magnetic fields [1]

$$i\phi_{1,t} + \phi_{1,xx} = (\overline{c_0} + \overline{c_2})(|\phi_1|^2 + 2|\phi_0|^2)\phi_1 + (\overline{c_0} - \overline{c_2})|\phi_{-1}|^2\phi_1 + 2\overline{c_2}\phi_{-1}{}^*\phi_0{}^2,$$
$$i\phi_{-1,t} + \phi_{-1,xx} = (\overline{c_0} + \overline{c_2})(|\phi_{-1}|^2 + 2|\phi_0|^2)\phi_{-1} + (\overline{c_0} - \overline{c_2})|\phi_1|^2\phi_{-1} + 2\overline{c_2}\phi_1{}^*\phi_0{}^2,$$
$$i\phi_{0,t} + \phi_{0,xx} = (\overline{c_0} + \overline{c_2})(|\phi_1|^2 + |\phi_{-1}|^2)\phi_0 + 2\overline{c_0}|\phi_0|^2\phi_0 + 2\overline{c_2}\phi_0{}^*\phi_1\phi_{-1},$$

where $\overline{c_0} = \overline{c_2} = \pm c\,(c > 0)$, admit bright soliton solutions, dark soliton solutions and rogue wave solutions which were obtained by using the inverse scattering method[1, 2, 3].

The Hirota bilinear method is known as one of powerful tools to construct exact solutions of integrable partial differential equations. Other than having an advantage in systematic algebraic construction of exact solutions compared with the inverse scattering method, one of advantages of the Hirota bilinear method is to clarify the relationship with other soliton equations such as the KP equation. However, in our knowledge, constructions of exact solutions of the integrable three components GP equations for $F = 1$ spinor BECs by he Hirota bilinear method is still missing.

In this presentation, we will show how to construct soliton solutions of the three component GP equation for $F = 1$ spinor BECs by using the Hirota bilinear method. This is joint work with Yuta Tanaka and Ken-ichi Maruno.

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