University of Glasgow | School of Mathematics & Statistics

Topical Geometry and Integrable Systems

4th-8th July 2011

Welcome to Glasgow

We hope you enjoy your visit to the city and also find the workshop scientifically rewarding. Please ask any of us if we can do anything to help or to make your visit more enjoyable.

Local organisers   Chris Athorne, Matthew England, Misha Feigin, Claire Gilson, Christian Korff, Jonathan Nimmo, Ian Strachan.

Tropical Geometry and Integrable Systems

The purpose of the conference is to bridge the gap between the developments of recent years made in pure mathematics and problems arising in the context of integrable systems.
    `http://www.maths.gla.ac.uk/island/island4/`

Scientific organising committee

Rei Inoue (Chiba University, Japan)
Diane Maclagan (University of Warwick, UK)
Tetsuji Tokihiro (The University of Tokyo, Japan)
Alexander Veselov (Loughborough University, UK)

External funding

The meeting is funded by grants from the EPSRC, London Mathematical Society, Edinburgh Mathematical Society and the Glasgow Mathematical Journal Trust.

Public lecture

In association with the workshop there will be a Public Lecture (Wednesday 6th, 6-7pm), given by Professor Junkichi Satsuma. This lecture is intended for interested members of the public and will be followed by a reception.
    `http://www.maths.gla.ac.uk/island/island4/Public_Lecture.html`.

Proceedings

We will be approaching the Editorial Committee of the AMS Contemporary Mathematics Series with the intention of publishing proceedings. More details, including submissions guidelines and deadlines, will be circulated to speakers and participants when we hear about the outcome of the submission to the AMS. Contact Ian Strachan (`ian.strachan@glasgow.ac.uk`) for more information.

About the University of Glasgow

The university, founded in 1451, is one of the seven ancient universities in the British Isles and is the four oldest in the English-speaking world. It has been on the current site since about 1870. Its most famous alumni include Lord Kelvin, who was Professor of Natural Philosophy from 1846-1899, economist Adam Smith and mathematician Colin Maclaurin. There are currently a total of about 23,000 undergraduate and postgraduate students.

Things to see in Glasgow

Wednesday afternoon is free and may be used for sightseeing in Glasgow. A few suggestions are given below and if you need any advice, please ask one of the local organisers.

Within walking distance

- University Museums: Hunterian Museum and Hunterian Art Gallery including Charles Rennie Mackintosh House. Admission to the Art Gallery and Museum are free. Normally there is a charge to visit Charles Rennie Mackintosh House but on Wednesday afternoons after 2.00pm admission is free.
  `http://www.hunterian.gla.ac.uk/visit/`

- Kelvingrove Art Gallery and Museum : Highly recommended, it is probably the best museum and art gallery in Scotland. It has a large collection of paintings, the most famous being Dali's "Christ of St John of the Cross". Free admission.
  `http://www.glasgowlife.org.uk/museums/our-museums/kelvingrove/`

- Botanical Gardens and Kibble Palace: "The Botanic Gardens is known internationally for its impressive glass houses and extensive tropical and temperate plant collections from around the world."
  `http://www.glasgow.gov.uk/en/Residents/Parks_Outdoors/Parks_gardens/`

In the city

- City bus tour: You can board at the university and get off and on at 24 places on the circular route.
  `http://citysightseeingglasgow.co.uk/`

- The easiest way to get to the city centre is by underground. From 'Hillhead SPT Subway' on Byres Road (see map on p4, between labels 3 and 4) and get off at Buchanan Street or Queen Street.
  `http://www.spt.co.uk/`

- Glasgow Cathedral, St Mungo's Museum and the Necropolis.
  `http://www.glasgowcathedral.org.uk/`

- Burrell collection: A little complicated to get to. About 40 mins by public transport see
  `http://www.glasgowlife.org.uk/museums/our-museums/burrell-collection/` for instructions.

Eating and drinking

Coffee and lunch breaks will take place each day in the common room on the ground floor of the Mathematics and Statistics Building. Lunch will be provided without charge. The conference dinner will be at a restaurant called Cail Bruich West (cail bruich means "eat well" in Gaelic) at 8pm on Thursday evening. Its location is indicated on the map on p4. We will cover the cost for the invited speakers, and the cost for other participants will be approximately £30-35 per head.

The area around the university, the West End of Glasgow, is famous for its large number of coffee shops, bars and restaurants. These are principally found on Byres Road, Great Western Road, Ashton Lane, its continuation Cresswell Lane and on Gibson Street. In the evening, you will have the opportunity to explore some of these. A (far from exhaustive) selection is given below in which the numbers refer to locations on the map. If you need any further advice please ask.
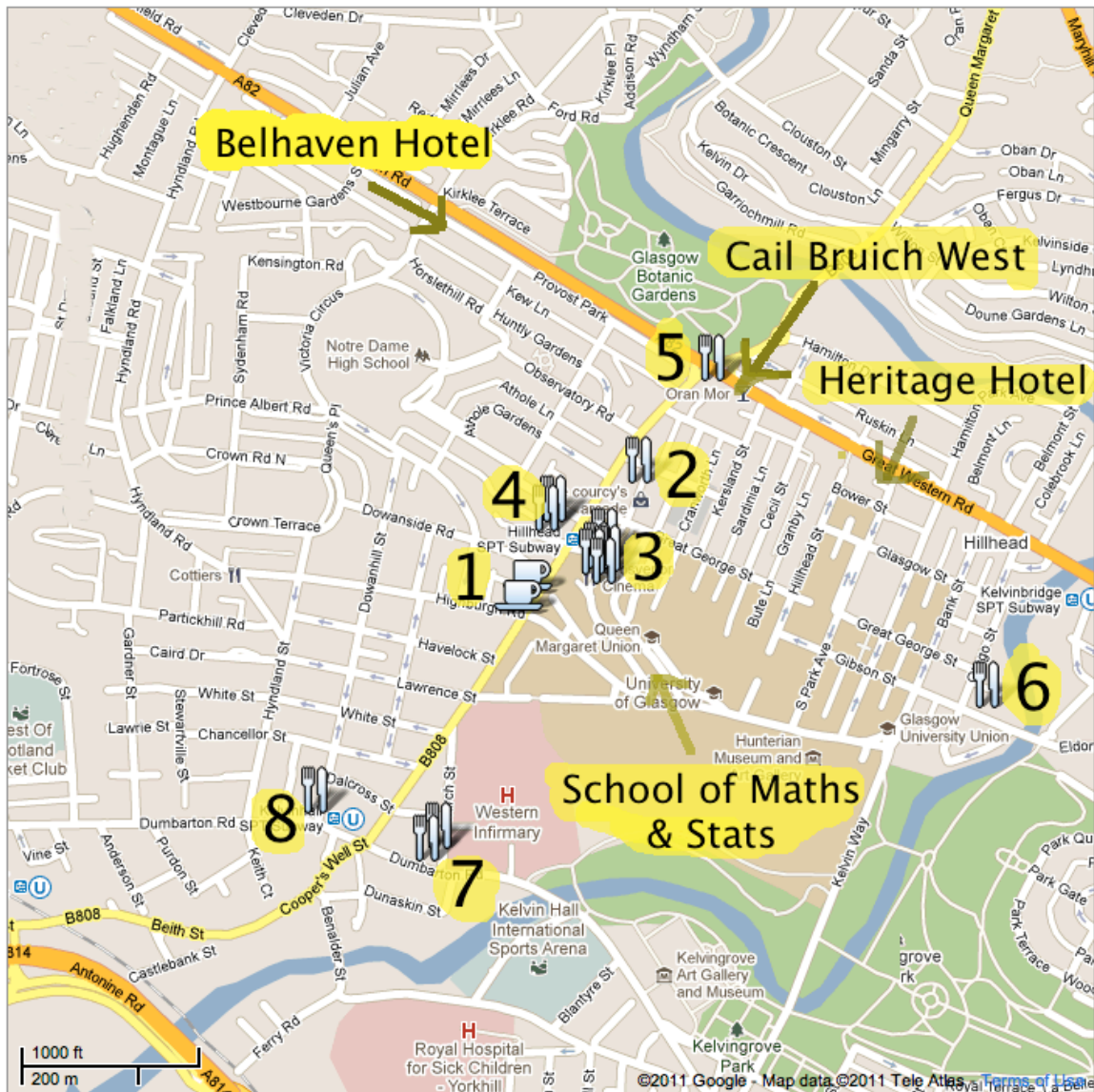
1. [Byres Road] Little Italy (Coffee); Tinderbox (Coffee)

2. [Cresswell Lane] Café Andaluz (Tapas)

3. [Ashton Lane] Ashoka (Indian); Brel (Belgian; wide range of Belgian beers too); Loft (Bar/restaurant upstairs); The Ubiquitous Chip (Famous restaurant downstairs, bar upstairs); The Wee Curry Shop (Indian, upstairs)

4. [Ruthven Lane] The Bothy (Scottish); Stravaigin 2 (Scottish); Da Maggio's (Italian)

5. [Byres Road] Òran Mór (Large bar serving food - former church)

6. [Gibson Street] Stravaigin (Scottish - worth the walk); the left bank (Frenchish)

7. [Church Street] Balbir's (Indian); [Dumbarton Road] Two Fat Ladies (Small fish restaurant; the restaurant is small but the fish come in many sizes)

8. [Dumbarton Road] Ichiban (Eclectic Japanese)

Workshop programme

	Monday 4th	Tuesday 5th	Wednesday 6th	Thursday 7th	Friday 8th
9:00	Registration and Coffee	Maclagan	Shapiro	Satsuma	Veselov
10:00	Maclagan	Sergeev	Kajiwara	Block	Johnson
10:45 / 11:00		Coffee	Coffee	Coffee	Coffee
11:15	Inoue	Inoue	Williams	Maeno	Fordy
12:00 / 12:15	Lunch	Lunch	Lunch	Lunch	END
13:30	Gaubert				
14:00		Draisma	Nakanishi	Mazzocco	
14:30	Tokihiro				
15:00		Willox	FREE	Iwao	
15:30 / 15:45	Coffee	Coffee		Coffee	
16:00 / 16:15	Q & A Session	Ikegami		Takagi	
17:00	Reception				
18:00			Public Lecture		
19:00			Reception	Conference Dinner	
20:00					

Venues	
Lecture room 515	All lectures. On the top floor of the Mathematics & Statustics Building.
Common room	Registration, coffee and lunch breaks, receptions. On the ground floor of the same building.
Cail Bruich West	Conference dinner (see map on p4)

Note

- Posters will be displayed in the Common Room throughout the conference.

- All times indicated in the left hand column are start times. Monday has a different schedule from the other days.

- The workshop ends on Friday, 8th at 12:00.

Abstracts

Speaker: Florian Block, University of Warwick

Title: Algebraic Curve Enumeration via Tropical Geometry

Abstract: Tropical geometry methods have been applied successfully to curve enumeration problems in algebraic geometry. A prominent connection between the two fields is Mikhalkin's "Correspondence Theorem," which replaces certain algebraic curve counts by tropical and, thus, more combinatorial enumerations.

In this expository talk, I will first explain how to "tropicalize" an algebraic curve enumeration problem. Then I will discuss how one can use the combinatorics of the tropical world to obtain new results in enumerative algebraic geometry.

Time: 45 mins

Notes

Speaker: Jan Draisma, Technische Universiteit Eindhoven

Title: Tropical Brill-Noether Theory

Abstract: A beautiful conjecture by Matthew Baker says that in a certain game with two players B and N, to be played with d chips and on a graph with first betti number g (edges minus vertices plus one), B has a winning strategy as soon as the numbers g, d, and a number r recording the strength of N satisfy the inequality $g - (r+1)(g-d+1) \geq 0$. The left-hand side in this inequality is motivated by an algebro-geometric analogue of this game, first studied by Brill and (Max) Noether, in which the graph is replaced by an algebraic curve. I will describe the interplay between the classical Brill-Noether theory and its graph (or "tropical") analogue. Apart from describing Baker's work and my own work with Cools, Payne, and Robeva, I will also touch on a very compelling "gonality conjecture" due to Castryck and Cools.

Time: 60 mins

Notes

Speaker: Allan Fordy, University of Leeds

Title: TBA

Abstract: TBA

Time: 45 mins

Notes

Speaker: Stéphane Gaubert, INRIA

Title: TBA

Abstract: TBA

Time: 60 mins

Notes

Speaker: Takatoshi Ikegami, Waseda University

Title: Max-plus Analysis of Elementary Cellular Automata

Abstract: Elementary cellular automaton (ECA) is a completely digital system defined by a binary evolution rule table. There are 256 different rules for ECA and they are classified into 4 classes about the asymptotic behavior of solution.

We discuss the initial value problem of ECA using the max-plus expression. The first step of our analysis is to express the evolution rule by the piecewise linear (max-plus) equation. The second is to solve the equation and derive the general solution in the max-plus form. The last is to evaluate the asymptotic behavior of the general solution. Through these steps, we extend the binary rule and solution to the real valued expression and utilize the various max-plus formulas.

Using the max-plus expression, we can introduce the viewpoint of analytics into the digital systems. Our goal is to revise the classification of ECA from this viewpoint.

This is joint work with **Daisuke Takahashi**.

Time: 45 mins

Notes

Speaker: Rei Inoue Yamazaki, Chiba University

Title: Tropical geometry and ultradiscrete integrable systems

Abstract: There is a nice class of integrable systems which go well with algebraic geometry in such a way that

   (i) the moduli space of the conserved quantities is given by a family of algebraic curves,

  (ii) the compactification of the general isolevel set is isomorphic to the Jacobian variety of the algebraic curve,

 (iii) the corresponding solution is written in terms of Riemann's theta function.

In this talk I introduce an application of tropical geometry to integrable systems, with the tropical (or ultradiscrete) version of the above story for the ultradiscrete periodic Toda lattice. I also give basic notions of tropical curve theory such as tropical Jacobian variety and tropical theta function. The box-ball system is also mentioned as the integrable cellular automaton closely related to ultradiscrete Toda lattice.

Time: 60 mins + 60 mins

Notes

Speaker: Shinsuke Iwao, Rikkyo University

Title: Tropical curve and periodic Box-Ball system

Abstract: An ultradiscrete integrable system is a combinatorial object which possesses a soliton-like behavior. In this talk, I introduce a two dimensional ultradiscrete integrable system with periodic boundary condition, which is called two dimensional periodic Box-Ball system (2dBBS). The dynamics of 2dBBS can be linearized on the Picard group of a tropical curve. An algorithm for calculating the fundamental cycle of 2dBBS is introduced.

Time: 45 mins

Notes

Speaker: Paul Johnson, Imperial College London

Title: Hurwitz numbers and Tropical Geometry

Abstract: Hurwitz numbers count maps between curves with specified ramification. Studying the monodromy of the map reduces their study to one about the symmetric group, where representation theory can find a lot of algebraic structure, including polynomiality and connections to integrable systems.

   Another approach, taken in joint work with Renzo Cavalieri and Hannah Markwig, is to tropicalize the maps of curves. This reduces calculating Hurwitz numbers to certain sums over weighted graphs. We will focus on this point of view, illustrating how it is suggestive of a conjecture of Goulden, Jackson and Vakil relating double Hurwitz numbers to some form of a universal Picard variety.

Time: 45 mins

Notes

Speaker: Kenji Kajiwara, Kyushu University

Title: Motion and Bäcklund transformations of the plane discrete curves

Abstract: We construct explicit solutions to discrete and continuous motions of discrete plane curves. Explicit formulas in terms the τ function are presented. Transformation theory of motions of both smooth and discrete curves is developed simultaneously. Continuous limits are also discussed.

Time: 45 mins

Notes

Speaker: Diane Maclagan, University of Warwick

Title: Introduction to tropical algebraic geometry

Abstract: These two expository lectures will form an introduction to tropical algebraic geometry. In the first lecture I will introduce the tropicalization of a subvariety of an algebraic torus, and state the Fundamental Theorem and Structure Theorem, which give alternate characterizations of tropical varieties, and describe some of their combinatorial properties. This will be followed in the second talk by connections to compactifications and degenerations, which involve connections to toric varieties.

Time: 60 mins + 60 mins

Notes

Speaker: Toshiaki Maeno, Kyoto University

Title: Sperner property, matroids and finite-dimensional Gorenstein algebras

Abstract: We discuss a combinatorial property of the vector space lattice and some polynomials associated to matroids. Stanley developed powerful methods based on Hard Lefschetz Theorem to handle combinatorial objects. It is known that the Sperner property of typical posets can be done by showing the Lefschetz property for related Artinian commutative graded algebras. Recently Numata and Watanabe proposed a new idea of a proof of the Sperner property for the vector space lattice based on the Lefschetz property for certain Artinian Gorenstein algebras. Their construction of the Artinian Gorenstein algebras is generalized to ones associated to matroids. We also discuss the Groebner fans of the defining ideals of our Gorenstein algebras and some tropical hypersurfaces.

Time: 60 mins

Notes

Speaker: Marta Mazzocco, Loughborough University

Title: Poisson algebras and symmetries of block-upper-triangular matrices

Abstract: Using the structure of algebroid of block-upper-triangular matrices composed from blocks of size $m \times m$ we obtain the Poisson brackets on the entries of these matrices and construct the braid-group action that preserves the Poisson algebra in the case of arbitrary m. We extend these algebras to semiclassical twisted Yangian algebras and find their central elements.

Time: 60 mins

Notes

Speaker: Tomoki Nakanishi, Nagoya University

Title: T-**systems,** Y-**systems, and cluster algebras**

Abstract: In the 90's the systems of discrete functional equations called T-systems and Y-systems were introduced and studied in the Bethe ansatz method for integrable models. After the introduction of cluster algebras by Fomin and Zelevinsky around 2000 it has been gradually recognized that T-systems and Y-systems are a part of cluster algebra structure. In particular the long standing conjecture of periodicities of Y- systems are proved by the tropicalization method in cluster algebras. As a further consequence, the associated classical and quantum dilogarithm identities are also obtained.

Time: 60 mins

Notes

Speaker: Junkichi Satsuma, Aoyama Gakuin University, Tokyo

Title: TBA

Abstract: TBA

Time: 60 mins

Notes

Speaker: Sergei Sergeev, INRIA

Title: Tropical two-sided systems

Abstract: This talk is an overview of theory, algorithms and applications of two-sided systems of equations (like $Ax = Bx$ or $Ax = By$) in tropical linear algebra and tropical convexity.

Time: 45 mins

Notes

Speaker: Michael Shapiro, State University of Michigan

Title: Poisson geometry of directed networks and integrable lattices

Abstract: A. Postnikov parametrized cells in positive Grassmannians using weighted directed planar graphs in a disk. We investigate Poisson properties of Postnikov's map from the space of edge weights of a planar directed network into the Grassmannian. We show that this map is Poisson if the space of edge weights is equipped with a representative of a 6-parameter family of universal quadratic Poisson brackets and the Grasmannian is viewed as an appropriate Poisson homogeneous space. Next, we generalize Postnikov's construction by defining a map from the space of edge weights of a directed network in an annulus into a space of loops in the Grassmannian. We use a special kind of directed networks in an annulus to study a cluster algebra structure on a certain space of rational functions and show that sequences of cluster transformations connecting two distinguished clusters are closely associated with Backlund-Darboux transformations between Coxeter-Toda flows in GL(n).

This is a joint work with M. Gekhtman and A. Vainshtein.

Time: 60 mins

Notes

Speaker: Taichiro Takagi, National Defence Academy, Japan

Title: Level set structure of a periodic box ball system

Abstract: I will talk about a one-dimensional cellular automaton associated with the periodic Toda lattice. It is known under the name of the periodic box-ball system (pBBS). One of the reasons why this system is attracting attentions is its relation with tropical algebraic geometry. Actually some recent studies report that the level set of the system is isomorphic to the tropical Jacobian of a tropical spectral curve. In my talk I will present a description of the level set from another viewpoint.

A commuting family of time evolutions is defined on the system based on Yang-Baxter relations in combinatorics of Young tableaux. It determines conserved quantities interpreted as solitons traveling along the system. Each level set of pBBS consists of all states of the system with a common soliton content. If we regard the states as nodes and the time evolutions as directed arcs, then the level set is a simple regular colored oriented graph.

I will show that every connected component of the graph has the structure of a torus. The non-linear dynamics of the cellular automaton is linearized on the torus as if it were a completely integrable system in Hamiltonian mechanics. The dimension, shape, and volume of each torus are specified by a matrix whose elements are determined by combinatorial Bethe ansatz for an integrable quantum spin chain.

Time: 45 mins

Notes

Speaker: Tetsuji Tokihiro, The University of Tokyo

Title: TBA

Abstract: TBA

Time: 60 mins

Notes

Speaker: Alexander Veselov, Loughborough University

Title: TBA

Abstract: TBA

Time: 60 mins

Notes

Speaker: Lauren Williams, University of California, Berkeley

Title: KP solitons, total positivity, and cluster algebras

Abstract: Soliton solutions of the KP equation have been studied since 1970, when Kadomtsev and Petviashvili proposed a two-dimensional nonlinear dispersive wave equation now known as the KP equation. It is well-known that the Wronskian approach to the KP equation provides a method to construct soliton solutions. More recently, several authors have focused on understanding the regular soliton solutions that one obtains in this way: these come from points of the totally non-negative (TNN) part of the Grassmannian.

In joint work with Yuji Kodama, we establish a tight connection between Postnikov's theory of total positivity for the Grassmannian, and the structure of regular soliton solutions to the KP equation. This connection allows us to apply machinery from total positivity to KP solitons. In particular, we completely classify the spatial patterns of the soliton solutions (which we call soliton graphs), coming from the totally positive (TP) part of $\mathrm{Gr}(2, n)$, as well as those coming from the TNN part of $\mathrm{Gr}(k, n)$, when the absolute value of the time parameter is sufficiently large. We also demonstrate an intriguing connection between soliton graphs for the TP part of $\mathrm{Gr}(k, n)$ and the cluster algebras of Fomin and Zelevinsky. We use this connection to solve the inverse problem for KP solitons coming from the TP part of $\mathrm{Gr}(k, n)$.

Time: 60 mins

Notes

Speaker: Ralph Willox, The University of Tokyo

Title: The ultradiscrete KdV equation defined over the real numbers

Abstract: I will explain how to solve the ultradiscrete KdV equation when the initial condition takes real (instead of integer) values. A linear problem and suitable "undressing" and dressing procedures are defined, which allow one to substract (add) a soliton from (to) a solution of the ultradiscrete KdV equation. I will show that a generic initial condition decomposes into a stable background (which may however include the slowest solitons) and solitons which asymptotically separate from this background.

These results show that the dynamics of the ultradiscrete KdV equation are quite rich and are almost on a par with those of its continuous and discrete counterparts.

Time: 45 mins

Notes

Posters

Posters will be displayed in the Common Room throughout the workshop.

Author: Kazuki Maeda, Kyoto University

Title: Box-ball systems and the nonautonomous finite Toda lattice

Author: Satyendra Nath Pandey, Motilal Nehru National Institute of Technology

Title: Integrability aspects of the general damped nonlinear oscillators and systems