A family of multi-value cellular automaton model for traffic flow

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Abstract

In this paper, a family of multi-value cellular automaton (CA) associated with traffic flow is presented. The family is obtained by extending the rule–184 CA, which is an ultradiscrete analogue to the Burgers equation. CA models in the family show both metastable states and stop-and-go waves, which are often observed in real traffic flow. Metastable states in the models exist not only on a prominent part of a free phase but also in a congested phase.

1 Introduction

Traffic phenomena attract much attention of physicists in recent years. It shows a complex phase transition from free to congested state, and many theoretical models have been proposed so far[1]–[7]. Among them we will focus on deterministic cellular automaton (CA) models. CA models are simple, flexible, and suitable for computer simulations of discrete phenomena.

The rule–184 CA[8] has been widely used as a prototype of deterministic model of traffic flow. In the model, lane is single and cars can move by one site at most every time step. Its several variations have been proposed recently: First, Fukui and Ishibashi (FI) proposed a high speed extension[6] of the rule–184 CA. In the model, cars can move more than one site per unit time if there are successive vacant spaces ahead. Second, Fukś and Boccara proposed a 'monitored traffic model'[9], which is a kind of quick start (QS) model. In the model, drivers can prospect vacant spaces due to car motion in the next site and can move more quickly compared with the rule–184 CA. Third, Takayasu and Takayasu proposed a slow start (SIS) model[10], in which cars can not move just after they stop and wait for a unit time. This represents an asymmetry of stopping and starting behavior. We call all these variants a family of rule–184 CA in this paper.

A desirable condition for CA as a traffic model is that it can show a phase transition observed in real data. Let us see an example of real data of Tomei expressway taken by the Japan Highway Public Corporation[12]. Fig.1 shows flow-density diagrams often called 'fundamental diagram', of both lanes in January 1996 on up line at 170.64 km point from Tokyo. Fig.1 (a) and (b) are diagrams of driving and acceleration lane respectively. We see that a phase transition from free to congested state around a density of 25 vehicles/km in both lanes, and there is a clearer discontinuity in the acceleration lane. The multiple states of flow at the same density around the critical density are also observed particularly in the acceleration lane, and a skeleton curve of plot points shows a shape of 'inverse λ '. These phenomena are often observed in real data[12].

There is another observed fact that over a certain critical density, a perturbation to a uniform traffic causes a formation of jam, and 'stop-and-go' wave propagates backward[13]. Cars accelerate and decelerate alternately in the wave. We adopt these two experimental facts, multiple states and stop-and-go wave, as criteria to judge whether a CA is suitable for traffic model or not. Among the rule–184 CA family, only SIS model shows multiple states and also shows a stop-and-go wave.

Recently, a multi-value generalization of the rule–184 CA has been proposed by using an ultradiscrete method[14]. Its evolution equation is

$$U_j^{t+1} = U_j^t + \min(U_{j-1}^t, L - U_j^t) - \min(U_j^t, L - U_{j+1}^t),$$
(1)

where U_j^t represents the number of cars at site j and time t, and L is an integer constant. Each site is assumed to hold L cars at most. We can prove that if L > 0 and $0 \le U_j^t \le L$ for any j, then $0 \le U_j^{t+1} \le L$ holds for any j. Thus (1) is considered to be (L+1)-value CA. Since (1) is obtained from an ultradiscretization of the Burgers equation, we call it Burgers CA (BCA)[15]. Note that BCA is equivalent to the rule-184 CA in a special case of L = 1. The positive integer L can be interpreted physically in the following three ways: First, the road is an L-lane freeway in a coarse sense, and effect of lane-changing rule is not expressed explicitly. In Appendix, we show BCA with L = 2 can be interpreted as a two lane model of the rule–184 CA with an explicit lane–changing rule. Second, U_j^t/L represents a probability distribution of cars in a single-lane freeway. In this case, U_j^t itself no longer represents a real number of cars at site j. Third, also in a single-lane freeway, length of each site is assumed to be long enough to contain L cars. By introducing the free parameter L into the rule–184 CA family, we can generalize them to multi-value CA and obtain rich algebraic properties and wide applications to various transport phenomena.

Though BCA does not show multiple states at a density in the fundamental diagram, we have shown that its high speed extension shows multiple states around a critical density[16]. Metastable states of flow in the model exist on a prominent part of a line in a free phase over a critical density. The model is also considered to be a multi-value generalization of FI model. This model is mentioned in Sec.2.3 to compare other models.

Other models in the rule–184 CA family, SlS model and QS model, are two-value CA like FI model. We generalize them to multi-value ones and investigate their properties as a traffic model in detail.

We use a max-plus representation to express time evolution rule of those generalized models. This representation comes from the ultradiscrete method which has a close relation with the max-plus algebra[11], as shown in our previous papers. Using max-plus representation, we can express evolution rule of the above CA's in a conservation from like (1), which automatically derives a conservation of total number of cars.

2 Generalization to multi-value CA

In this section, we will present four CA models which are multi-value generalization of the rule–184 CA family. Before explaining models, we shortly review a rule of car movement of BCA. Note that we assume L is a lane number of the road in the description of all models. In BCA, cars at site j move to vacant spaces at their next site j + 1 as many as possible. Therefore, car flow from j to j + 1 at time t becomes $\min(U_j^t, L - U_{j+1}^t)$. Thus adding $\min(U_{j-1}^t, L - U_j^t)$ (flow from j - 1 to j) and $-\min(U_j^t, L - U_{j+1}^t)$ (flow from j to j + 1) to U_j^t (present car number), we obtain U_j^{t+1} and derive (1).

2.1 Multi-value QS model

First, we generalize QS model to a multi-value one. In the multi-value model, cars at site j move to vacant spaces at site j + 1 per unit time as many as possible. This movement rule is similar to that of BCA. However, a difference is that drivers at j estimate vacant spaces at j + 1 by predicting how many cars move from j + 1 to j + 2. Thus, they estimate vacant spaces at j + 1 at time t to be $L - U_{j+1}^t$ (present spaces) + $\min(U_{j+1}^t, L - U_{j+2}^t)$ (predicted spaces due to BCA movement). Therefore considering the number of cars coming into and escaping from site j, evolution equation on U_j^t is given by

$$U_{j}^{t+1} = U_{j}^{t} + \min(U_{j-1}^{t}, L - U_{j}^{t} + \min(U_{j}^{t}, L - U_{j+1}^{t})) - \min(U_{j}^{t}, L - U_{j+1}^{t} + \min(U_{j+1}^{t}, L - U_{j+2}^{t})) = U_{j}^{t} + \min(U_{j-1}^{t}, 2L - U_{j}^{t} - U_{j+1}^{t}) - \min(U_{j}^{t}, 2L - U_{j+1}^{t} - U_{j+2}^{t}).$$
(2)

The final form of (2) means that the number of movable cars (U_j^t) is limited by vacant spaces $(2L - U_{j+1}^t - U_{j+2}^t)$ in their next two sites. In the case of L = 1, this model is identical to QS model proposed by Fukś and Boccara, and it is the rule-3212885888 CA of a neighborhood size '5' after Wolfram's terminology[8].

2.2 Multi-value SIS model

In the original SIS model, lane is single and cars can not move just after they stop and wait for a unit time. Movable cars move to vacant spaces in their next site like BCA. In the multi-value case, we should distinguish standing cars and moving cars in each site because each site can hold plural cars. The number of cars at site j blocked by cars at j + 1 at time t - 1 is represented by $U_j^{t-1} - \min(U_j^{t-1}, L - U_{j+1}^{t-1})$. These cars cannot move at t, then the maximum number of cars movable to site j at t is given by $U_j^t - \{U_j^{t-1} - \min(U_j^{t-1}, L - U_{j+1}^{t-1})\}$. Therefore, the multi-value SIS CA is given by

$$U_{j}^{t+1} = U_{j}^{t} + \min\left[U_{j-1}^{t} - \{U_{j-1}^{t-1} - \min(U_{j-1}^{t-1}, L - U_{j}^{t-1})\}, L - U_{j}^{t}\right] - \min\left[U_{j}^{t} - \{U_{j}^{t-1} - \min(U_{j}^{t-1}, L - U_{j+1}^{t-1})\}, L - U_{j+1}^{t})\right]$$
(3)

We note that this model includes the original SIS model in the case L = 1. Since U^{t+1} is determined by U^t and U^{t-1} , the evolution equation is second order in time and an effect of inertia of cars is included in this model.

2.3 EBCA2 model

In our previous paper[16], we propose an extended BCA (EBCA) model in which cars can move two site forward at a unit time if the successive two sites are not fully occupied. In this paper, we call the model 'EBCA2' and another model with a similar extension 'EBCA1' described in the next subsection. A main difference between two models is a priority of movement of fast and slow cars to vacant spaces. Fast cars with speed 2 move prior to slow ones with speed 1 in EBCA2 and otherwise in EBCA1.

Evolution equation of EBCA2 is given by

$$U_{j}^{t+1} = U_{j}^{t} + \min(b_{j-1}^{t} + a_{j-2}^{t}, L - U_{j}^{t} + a_{j-1}^{t}) - \min(b_{j}^{t} + a_{j-1}^{t}, L - U_{j+1}^{t} + a_{j}^{t}),$$
(4)

where $a_j^t \equiv \min(U_j^t, L - U_{j+1}^t, L - U_{j+2}^t)$ which is the number of cars moving by two sites and $b_j^t \equiv \min(U_j^t, L - U_{j+1}^t)$. In reference [16], we only study the cases of L = 1 and 2. In this paper, we show some properties of flow for L in the next section. Note that (4) in the case of L = 1 is FI model which is equivalent to the rule-3436170432 CA.

2.4 EBCA1 model

There is another possibility when we extend BCA to a high-speed model and we call this extended new model EBCA1. In contrast to EBCA2, slow cars with speed 1 move prior to fast ones with speed 2. Then car movement from t to t + 1 consists of the following two successive procedures,

a) Cars move to vacant spaces in their next site as many as possible.

b) Only cars moved in procedure a) can move more one site and they move to their next vacant spaces as many as possible.

The number of moving cars at site j and time t in procedure a) is given by $b_j^t \equiv \min(U_j^t, L - U_{j+1}^t)$. In procedure b), the number of moving cars at site j+1 becomes $\min(b_j^t, L - U_{j+2}^t - b_{j+1}^t + b_{j+2}^t)$, where the second term in min represents vacant spaces at site j+2 after the first procedure a). Therefore, considering a total number of cars entering into and escaping from site j, evolution equation of EBCA1 is given by

$$U_{j}^{t+1} = U_{j}^{t} + b_{j-1}^{t} - b_{j}^{t} + \min(b_{j-2}^{t}, L - U_{j}^{t} - b_{j-1}^{t} + b_{j}^{t}) - \min(b_{j-1}^{t}, L - U_{j+1}^{t} - b_{j}^{t} + b_{j+1}^{t}) = U_{j}^{t} + \min(b_{j-1}^{t} + b_{j-2}^{t}, L - U_{j}^{t} + b_{j}^{t}) - \min(b_{j}^{t} + b_{j-1}^{t}, L - U_{j+1}^{t} + b_{j+1}^{t}).$$
(5)

Note that (5) with L = 1 differs from FI model and it is equivalent to rule-3372206272 CA.

We can consider that this rule is an extension of SIS model to high-speed motion since cars which stop at procedure a) cannot move at procedure b). Let us consider a relation between SIS and EBCA1 models in detail. If we write each procedure of EBCA1 explicitly using an intermediate time step, we obtain

$$U_{j}^{t+1/2} = U_{j}^{t} + b_{j-1}^{t} - b_{j}^{t}$$

$$U_{j}^{t+1/2} = U_{j}^{t+1/2} + \min(U_{j-1}^{t+1/2} - \{U_{j-1}^{t} - b_{j-1}^{t}\}, L - U_{j}^{t+1/2})$$

$$-\min(U_{j}^{t+1/2} - \{U_{j}^{t} - b_{j}^{t}\}, L - U_{j+1}^{t+1/2}).$$
(6)
(7)

Equations (6) and (7) represent above procedure a) and b) respectively, and $U_j^{t+1/2}$ denotes car number at site j just after procedure a). If we replace t+1 by t+1/2 in (1), we obtain (6). Moreover, if we replace t by t+1/2 and t-1 by t in (3), we obtain (6). Therefore we can consider EBCA1 to be a 'combination' of BCA and SIS rules.

3 Fundamental diagram and multiple states

We discuss fundamental diagrams of new CA models described in the previous section. In the followings, we will consider a periodic road or a circuit. All models in Sec.2 can be expressed in a conservation form such as

$$\Delta_t U_j^t + \Delta_j q_j^t = 0, \tag{8}$$

where Δ_t and Δ_j are forward difference operator with respect to indicated variable, and q_j^t represents a traffic flow. Average density ρ and average flow Q^t over all sites are defined by

$$\rho \equiv \frac{1}{KL} \sum_{j=1}^{K} U_j^t, \qquad Q^t \equiv \frac{1}{KL} \sum_{j=1}^{K} q_j^t, \tag{9}$$

where K is number of sites in a period. Since all models are in a conservation form, average density does not depend on time and we can use ρ without a script t.

Figures 2 (a)–(d) are density–flow diagrams of previous models with L = 2 and K = 30. Figures 2 (a)–(d) corresponds to multi-value QS, multi-value SIS, EBCA2 and EBCA1

models respectively. If we set a number N of cars, we obtain a unique density $\rho = N/KL$ but there are many initial distributions of cars. Each initial distribution does not always reach a steady flow and often make a periodic state. Q^t can also changes periodically in time. Therefore, we plot every points in figures by averaging Q^t from t = 2K to t = 4K. (We use Q as an average flow.) Note that t = 2K is long enough from initial time to obtain a periodic state and 2K time steps is much longer than its period.

Moreover, we obtain different values of Q from different initial distributions for a given ρ in Figs.4 (b) ~ (d). In Ref.[16], this type of state giving different Q values for the same density are called 'multiple states'[16]. We use this terminology for other models in this paper. Multiple states exist around a critical density. Especially for multi-value SIS model (Fig.2 (b)) and EBCA1 model (Fig.2 (d)), we see a thick branch other than lines forming an inverse λ shape and the distribution of data in the branch look random. This fact is interesting because evolution rules are completely deterministic.

Next, we focus on EBCA1 model and examine its properties in detail. Figure 3 is a skeleton diagram of EBCA1 model. Branches in Fig.3 are drawn as straight lines and we obtain them using specific initial conditions. For example, states ' \cdots 11102021110202 \cdots ', ' \cdots 121212 \cdots ', ' \cdots 002002 \cdots ' and ' \cdots 222222 \cdots ' are all steady states in the model, and flows of these states are plotted on B, C, D and E respectively in Fig.3. There are two branches in congested phase of higher density, that is, B–C and D–E. We call the branch D–E and B–C congested phase zero and phase one respectively. It is noted that there are many states not on the skeleton in Fig.3. For example, state \cdots 211211 \cdots is a steady state of the model, and it has density 2/3 and flow 1/2.

Fig.4 shows a time evolution of flow Q^t in EBCA1 model with a density of phase transition region. Number of total sites K is 60 and 240 in Fig.4 (a) and (b) respectively. In the figures, we can see periodic oscillation of flow appearing soon after an initial time. We observe that the maximum period of oscillation becomes longer as system size becomes larger. Moreover, oscillation in a period is not simple as shown in the figures. Fig.4 (c) shows a power spectrum I versus frequency f defined by $I = |\sum_{t=0}^{T-1} Q^t \exp(2\pi i f t/T)|/T$. We set K = 240 and T = 1000. This indicates that the irregular oscillation is similar to white noise.

Fig.5 is a fundamental diagram of EBCA1 with L = 1. It is interesting that there exist multiple states even in the case L = 1, which is equivalent to the rule–3372206272 CA. Among deterministic two-value CA models, only SlS model is known to show metastable state so far. Thus, EBCA1 with L = 1 becomes the second example of such kind of two-value CA.

At the end of this section, we give a comment on a case of L > 3. Fundamental diagram of multi-value QS model is the same regardless of L. In other models, small branches increases around the critical density as L becomes larger. Let us show this phenomenon using EBCA2 which has clear branches. The fundamental diagram of EBCA2 with L = 7is given in Fig.6. There are many branches around the critical density. We can give a partial explanation of these branches. Let us assume initial values on all sites are restricted to n or L-n where $0 \le n < L/2$. Then, if we use a transformation V = (U-n)/(L-2n)from U to V, V = 0 corresponds to U = n and V = 1 to U = L - n. We can easily show that evolution equation on V is obtained by replacing U by V and L by 1 in (4). Equation (4) with L = 1 is FI model itself and it is two-value CA. Therefore, if all initial values of U is restricted to n or L - n, then U at any site is always n or L - n and evolutional state is the same as that of FI model by replacing 0 by n and 1 by L - n.

Above fact implies a shape of the fundamental diagram. Consider an arbitrary state of FI model, that is, (4) with L = 1. Let us assume density is ρ and average flow is Q for that state. We can obtain a corresponding state of (4) with L > 0 by replacing 0 by n and 1 by L - n. Then, using (4), we can easily show that density of that state is $(1 - \frac{2n}{L})\rho + \frac{n}{L}$ and flow is $(1 - \frac{2n}{L})Q + \frac{2n}{L}$. Moreover, we can also show that a fundamental diagram of FI model is exactly given by two segments of which end points are (0,0), $(\frac{1}{3},\frac{2}{3})$ and $(\frac{1}{3},\frac{2}{3})$, (1,0) respectively. Therefore, a diagram of EBCA2 with L > 0 has at least a segment with end points $(\frac{n}{L},\frac{2n}{L})$ and $(\frac{1}{3} + \frac{n}{3L},\frac{2}{3} + \frac{2n}{3L})$ and $(1 - \frac{n}{L},\frac{2n}{L})$ for any n ($0 \le n < L/2$). Figure 6 is a L = 7 case and the shape of diagram is strictly a superposition of all these segments.

4 Stability of flow

In this section, we investigate stability of the branches in fundamental diagram obtained in the previous section. First, we study a uniform state $\cdots 11111\cdots$, which gives the maximum flow in each model. Let us define a weak perturbation by a perturbation changing a local state 11 to 20, and a strong perturbation by a perturbation changing 1111 to 2200. Both perturbations clearly do not change the density. These perturbations mean that a car or two cars in the uniform flow suddenly decrease their speed and consequently fully-occupied sites '2' appear.

Figures 7 (a)–(c) show an instability of uniform flow by a weak perturbation in multivalue SIS, EBCA2 and EBCA1 models respectively. We can observe a stop-and-go wave propagating backward from the perturbed site in Fig.7 (a) (multi-value SIS) and (c) (EBCA1), while the wave does not appear in (b) (EBCA2). In Fig.7 (c), flow Q decreases by the weak perturbation and transits from A to F in Fig.3. Moreover, we see that the final steady state contains locally two states corresponding to B and C in Fig.3. Flow of state B is 5/6 and that of C is 1/2, both states are in the phase one, and they give the maximum and minimum flow in that phase. Numerical results from various initial states with a weak perturbation shows that a state of higher density tends to give this type of extreme local states on a branch after some time steps. Moreover, if we give a strong perturbation of uniform state ... 111... corresponding to A, it goes down directly to G(1/2,1/2) in the phase zero. Fig.7 (d) shows an instability due to a strong perturbation in EBCA1 model. In this case, stop-and-go wave does not occur. Moreover, we see that the final state consists of two local states $\cdots 200200\cdots$ and $\cdots 222222\cdots$ corresponding to D and E in Fig.3, which also give the maximum and minimum flow in the phase zero respectively.

Since a perturbed uniform state becomes a congested one which consists of states giving the maximum and the minimum flow in a phase, we can predict a length of the final congested bunch. Assume α denotes a ratio of length of final congested bunch to length of total sites. Density of B, F and C are 5/12, 1/2 and 3/4 respectively. Thus we obtain

$$\frac{5}{12}(1-\alpha) + \frac{3}{4}\alpha = \frac{1}{2},$$

and α becomes 1/4 which coincides with that of Fig.7 (c). In the case of the phase zero, we also obtain $\alpha = 1/4$ (Fig.7(d)).

Contrary to above facts, there also exist some states on branches stable against a weak perturbation. Let us consider states on branch A–D in Fig.3. There exists a state $\cdots 011011011 \cdots$ on the branch and we obtain $\cdots 011020011 \cdots$ by a weak perturbation. Against the perturbation, perturbed state is stable and it is still on the branch A–D. We call this type of state metastable state. Moreover, let us consider a state $\cdots 111120111120 \cdots$ corresponding to F in Fig.3. We can obtain $\cdots 201120111120 \cdots$ by a weak perturbation, and its flow does not change. Therefore, metastable states exist not only on branch A–D but also on the phase one branch. In these cases, the effect of the site "2" does not spread over the entire system during the time evolution. Thus some part of the multiple branches are metastable states, which can be observed in long time simulations starting from random initial conditions.

5 Concluding discussions

In this paper, some multi-value generalizations of rule–184 family are studied by using max–plus representation. We obtain evolution equations in a conserved form giving (L + 1)-value CA. We have proved that EBCA1 model is a generalization of SIS model, and it gives multiple states and stop-and-go waves. We consider that a suitable traffic model gives those two phenomena at least. If we observe a real traffic data like Fig.1, existence of multiple states is clear. In a real traffic, cars are considered to be always perturbed by some traffic effects. Therefore, we consider existence of stable branches against a weak perturbation is also desirable for a model. Moreover, stop-and-go waves are often observed in a real congested traffic. Considering these points, EBCA1 is the most suitable model among the models of this paper.

Next, we give a presumption on a fundamental diagram of real data. In Fig.1, we see the multiple state clearly in an acceleration lane. In a driving lane, since the average speed of cars is slower than the acceleration lane, flow tends to be stable and the branch is not clear. In the acceleration lane, drivers move faster than cars in the driving lane, then over-dense free flow will be likely to occur and we can see the multiple state around a critical density in the diagram. Moreover, from the stability analysis we find that there are metastable states even in the congested state in the EBCA1 model. Although we cannot clearly see the fact due to fluctuation of data in Fig.1, we expect that there exists both a metastable "weak jam" (line B–C) and a "strong jam" (line D–E) in a real highway traffic.

Finally, we point out future problems. We have investigated mainly about models with L = 2. About multiple branches due to larger L, only EBCA2 gives clear branches and we can easily discuss their stability. Though other models can also make multiple branches, transformation of perturbed states is complicated and its analysis is difficult. We should solve this difficulty because we consider EBCA1 model is more suitable for a traffic model. Moreover, two-dimensional extension of these models and its application to a real traffic network is one of the important future problems.

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Appendix

In this appendix, we show that BCA model with L = 2 can be interpreted as a two-lane model of rule-184 CA. Let us call one of the two lanes A-lane and the other B-lane. Both lanes are divided into discrete sites as shown in Fig.A1. Assume A_j^t and B_j^t denote number of cars at site j and time t in A-lane and B-lane respectively. Since all sites can hold only one car at most, A_j^t and B_j^t are always 0 or 1. A car at site j in A-lane moves according to the following rule:

- (a) If site j + 1 in A-lane is empty, the car moves to that site.
- (b) If site j + 1 in A-lane is not empty and site j and j + 1 in B-lane are both empty, the car can move to site j + 1 in B-lane.
- (c) Otherwise, the car stays at site j in A-lane.

As for a car in B-lane, symmetrical rule with respect to lane symbol is applied. The above rule can be interpreted as follows: Every car move in its own lane prior to the other lane (rule (a)). However, if a car can not move in its own lane and only if there are no traffic in the other lane, it changes a lane and move forward in the other lane (rule (b)). Since cars move independently in each lane according to rule–184 if rule (b) is omitted, the above rule can be interpreted as a two-lane model based on rule–184 with lane-changing effect.

We can express the rule by a couple of evolution equations as follows:

$$\begin{aligned}
A_{j}^{t+1} &= A_{j}^{t} + \min(A_{j-1}^{t}, 1 - A_{j}^{t}) - \min(A_{j}^{t}, 1 - A_{j+1}^{t}) \\
&+ \min(1 - A_{j-1}^{t}, 1 - A_{j}^{t}, B_{j-1}^{t}, B_{j}^{t}) - \min(A_{j}^{t}, A_{j+1}^{t}, 1 - B_{j}^{t}, 1 - B_{j+1}^{t}), (10) \\
B_{j}^{t+1} &= B_{j}^{t} + \min(B_{j-1}^{t}, 1 - B_{j}^{t}) - \min(B_{j}^{t}, 1 - B_{j+1}^{t}) \\
&+ \min(1 - B_{j-1}^{t}, 1 - B_{j}^{t}, A_{j-1}^{t}, A_{j}^{t}) - \min(B_{j}^{t}, B_{j+1}^{t}, 1 - A_{j}^{t}, 1 - A_{j+1}^{t}). (11)
\end{aligned}$$

The last two terms in both equations express lane-changing effect. If those terms are omitted, both equations become independent each other and are equivalent to rule-184 CA (BCA model with L = 1). If U_i^t is defined by

$$U_j^t = A_j^t + B_j^t, (12)$$

it denotes a sum of cars of both lanes. Since A_j^t and B_j^t is always 0 or 1, we can derive an evolution equation on U_j^t from (10) and (11) as follows:

$$U_j^{t+1} = U_j^t + \min(U_{j-1}^t, 2 - U_j^t) - \min(U_j^t, 2 - U_{j+1}^t).$$
(13)

This equation is equivalent to BCA with L = 2.

Figure Captions

- Fig.1 Observed data of flow (vehicles/5min.) versus density (vehicles/Km) on Tomei expressway. This diagram was taken by Japan Highway Public Corporation. (a) driving lane, (b) acceleration lane.
- Fig.2 Fundamental diagrams of multi-value CA models with L = 2 and K = 30: (a) multi-value QS, (b) multi-value SIS, (c) EBCA2, (d)EBCA1 models.
- Fig.3 Schematic fundamental diagram of EBCA1 model.
- Fig.4 Time evolution of flow in EBCA1 model. (a) K = 60, (b) K = 240, (c) power spectrum I versus frequency f of traffic flow for K = 240.
- Fig.5 Fundamental diagram of EBCA1 model with L = 1.
- Fig.6 Fundamental diagram of EBCA2 model with L = 7.
- Fig.7 Instability of uniform flow due to a weak perturbation. (a) multi-value SIS, (b) EBCA2, (c) EBCA1 models. (d) shows an instability due to a strong perturbation in EBCA1 model. In all figures, black, dark gray, light gray squares denote a value 2, 1, 0 respectively.
- Fig. A1 Interpretation of BCA with L = 2 as a model of coupled single lanes.

Figure 1 by K. Nishinari and D. Takahashi















Figure 3 by K. Nishinari and D. Takahashi





Figure 4(a) by K. Nishinari and D. Takahashi



Figure 4(b) by K. Nishinari and D. Takahashi



Figure 4(c) by K. Nishinari and D. Takahashi













Figure A1 by K. Nishinari and D. Takahashi

	Aj-1	Aj	Aj+1		
	Bj-1	Bj	Bj+1		