Metastable Flows in A Two-Lane Traffic Model Equivalent to Extended Burger's Cellular Automaton

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Abstract. A two-lane cellular automaton traffic model equivalent to the extended Burger's cellular automaton has been proposed, and evolution equations for cars on the two-lane road are obtained. Configuration of cars on the road are simulated by using the equations and many metastable local congested states are found in two-dimensional region on density-flow diagram. There are three typical states in the congested states: The first is that cars advance by stop-and go-flow on their own lanes without lane-change and values of the flow are stable in time. The second is that cars change the lane periodically with several time-steps. The third is that they advance changing the lane that induces fluctuation of the flow with extremely long period. This fluctuation flow exists in wide range between car densities 5/12 and 3/4. The metastable states are discussed in connection with the synchronized states observed in the traffic flow on expressway.

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1. Introduction

Recently traffic problems have attracted a great interest of many physicists [1-3]. Traffic flow is a kind of many-body systems of strongly interacting cars and occurrence of traffic jams can be regarded as a kind of phase transition. Traffic flow on expressways has been usually studied by the one-dimensional traffic models such as car-following models[4-7] and cellular automaton (CA) models[8-12]. Prototype of CA models for traffic flow is the rule-184 model proposed by Wolfram[13]. The rule-184 model, where the lane is single and cars can move by one site at one time step, has been extended to various models, e.g., a high speed model[14], a quick start model[15], a slow start model[16], bottleneck models[17-20], two-lane models[21-24], cross road model[25,26] and two-dimensional models[11,27-28].

More recently, a new CA traffic model[29,30] has been proposed as a transformation of the Burger's equation by using the ultradiscrete method[31], which is called Burger's CA (BCA). The BCA model is a discrete model derived from the continuous equation of motion and holds physical properties contained in the original equation. The ultradiscrete method is quite nice as a method to elucidate the physical background of CA model. The BCA can also be regarded as a multi-value generalization of the rule-184 CA. The multiple states in the BCA can be interpreted to represent multi-lane expressway in traffic model. The BCA is further extended to several CA models by combination with a high-speed model, a quick start model and a slow start model. In these extended BCA models, authors are especially interested in the

EBCA1 model proposed by Nishinari and Takahashi[32], where the maximum velocity of car is extended to "2" and a slow start model is included. EBCA1 can be concluded as a multi-value (multi-lane) and multi-neighbor (high-speed) extension model of the rule-184. The most notable character of this model is an appearance of new branches of metastable states; i.e. metastable free flow states and metastable congested states around the critical density in the density-flow diagram.

It is known that cars advancing on different lanes of the expressway is synchronized each other and velocities of the cars become nearly the same[33]. In recent years a lot of traffic data of many German expressways are accumulated and characteristic features of the traffic are analyzed by Kerner and Rehborn[34-37]. They have focused on synchronized flow and proposed that there are three phases in the complex traffic flow: free flow, synchronized flow and wide moving jam[34]. The jams usually emerge in free flow through a sequence of two phase transitions: "free flow→ synchronized flow→jams". The direct transition from free flow to jam occurs only if the formation of synchronized flow is strongly hindered[37]. They moreover say that the synchronized flow is composed of three kinds of phases[35]. Thus the synchronized phase becomes an essential key word to investigate the traffic flow and then it is indispensable to elucidation of traffic flow.

Desirable conditions for CA traffic model are that (1) it can show a phase transition from the free flow to the congested state, (2) the flow-density diagram shows a shape of 'inverse λ', (3) the multiple states of flow exist around the critical density and (4) the stop-and-go wave exists in congested states, which are all observed in real data from the expressways. EBCA1 is a model satisfying these conditions.

In the present paper, a two-lane traffic model equivalent to the EBCA1 is proposed in Section 2. Traffic flow is studied in detail in Section 3. Many metastable branches in fundamental diagram are analyzed. In Section 4 configurations of cars on each lane in the congested states and lane-changing phenomena are investigated numerically, and dynamics of cars is discussed in connection with the synchronized states in Section 5.

2. Extended Burger's Model (EBCA1) and Equivalent Traffic Model

In EBCA1, cars advance by following two successive procedures.

a) Cars move to their next site if the site is not fully occupied.

b) Only cars moved in the procedure a) can move one more site if their next site is not fully occupied after the procedure a).

The procedure b) expresses the slow start rule where cars that could not move in the procedure a) stay at that site even if their next site is not fully occupied. The velocity is extended to 2 through the procedures a) and b).

These procedures have been expressed by following an evolution equation[32].

$$U_j^{t+1} = U_j^t + b_{j-1}^t - b_j^t$$
$$+ \min(b_{j-2}^t, L - U_j^t, - b_{j-1}^t + b_j^t) - \min(b_{j-1}^t, L - U_{j+1}^t - b_j^t + b_{j-1}^t),$$
(1)

where U_j^t ($\in \{Z \mid 0 \leqq U_j^t \leqq L\}$) is the number of states at the site j and time t. L is assumed as a capacity in each site. The number of moving cars at site j and time t by procedure a) is given by $b_j^t \equiv \min(U_j^t, L-U_{j+1}^t)$. The second term in the right-hand side of eq.(1) is the number of cars coming from the site j-1 and the third term going out to j+1. In procedure b), the number of moving cars at site j becomes $\min(b_{j-1}^t, L-U_{j+1}^t-b_j^t\ b_{j+1}^t)$, where the second term in min represents vacant spaces at site j+1 after the procedure a).

The multiple state expresses that each site can hold L cars at maximum. Now the multiple states are interpreted as a multiple-lane model. Let us consider the case L=2 in this paper, i.e. two-lane model, which are named A- and B-lane. Both lanes are divided into discrete sites as shown in Fig.1. A_j^t and B_j^t denote numbers of cars at site j and time t in the A-lane and B-lane, respectively and hence A_j^t and B_j^t are always 0 or 1. Above procedures a) and b) are rewritten for each lane as follows. For procedure a), a car at site j in A-lane move according to the following rule:

If site j+1 in A-lane is empty, the car moves to that site.

(a) If site j+1 in A-lane is occupied and sites j and j+1 in B-lane are both empty, the car moves to site j+1 in B-lane.

(b) Otherwise, the car stays at site j in A-lane.

As for a car in B-lane, symmetrical rule with respect to lane symbol is applied. For procedure b), the following rules (d)~(e) are applied to only cars moved by rules (a) and (b). For the moved car in site j+1,

(c) If site j+2 in A-lane is empty, the car moves to that site.

(d) If site j+2 in A-lane is not empty and site j+2 in B-lane is empty and site j+1 in B-lane is empty or occupied by the stopped car, then the car moves to site j+2 in B-lane.

(e) Otherwise, the car stays at site j+1 in A-lane.

Rules (d)~(f) are the same as rules (a)~(c) except that an effect of the stopped car by the rule (c) is taken into account. Rules (b) and (e) include a lane-changing rule. An example of moving of cars is shown in Fig.1, where a) and b) shows procedure a) and b), respectively.

When the variable U_j^t is denoted by the sum of cars of both lanes

$$U_j^t = A_j^t + B_j^t , \qquad (2)$$

we can rewrite eq.(1) separately to two equations for cars on A and B-lanes, considering rules (a)～(f), as follows:

$$\begin{aligned} A_j^{t+1} = A_j^t &+ \min(A^*{}_{j-1}^t, 1-A_j^t) - \min(A^*{}_j^t, 1-A_{j+1}^t) \\ &+\min(1-A^*{}_{j-1}^t, 1-A_j^t, B^*{}_{j-1}^t, B_j^t) \\ &-\min(A^*{}_j^t, A_{j+1}^t, 1-B^*{}_j^t, 1-B_{j+1}^t), \end{aligned} \qquad (3)$$

$$\begin{aligned} B_j^{t+1} = B_j^t &+ \min(B^*{}_{j-1}^t, 1-B_j^t) - \min(B^*{}_j^t, 1-B_{j+1}^t) \\ &+\min(1-B^*{}_{j-1}^t, 1-B_j^t, A^*{}_{j-1}^t, A_j^t) \\ &-\min(B^*{}_j^t, B_{j+1}^t, 1-A^*{}_j^t, 1-A_{j+1}^t), \end{aligned} \qquad (4)$$

where

$$\begin{aligned} A^*{}_j^t &= A_j^t - [A_j^{t-1} - \min(A_j^{t-1}, 1-A_{j+1}^{t-1}, \max(1-A_j^t, 1-A_{j+1}^t)) \\ &\quad -\min(A_j^{t-1}, A_{j+1}^{t-1}, 1-B_{j+1}^{t-1}, \max(1-B_j^{t-1}, 1-A_j^t))], \end{aligned} \qquad (5)$$

$$\begin{aligned} B^*{}_j^t &= B_j^t - [B_j^{t-1} - \min(B_j^{t-1}, 1-B_{j+1}^{t-1}, \max(1-B_j^t, 1-B_{j+1}^t)) \\ &\quad -\min(B_j^{t-1}, B_{j+1}^{t-1}, 1-A_{j+1}^{t-1}, \max(1-A_j^{t-1}, 1-B_j^t))]. \end{aligned} \qquad (6)$$

The last two terms in the first line in eqs.(3) and (4) express the number of cars going ahead on their own lane, and the last two lines show that of cars changing lane. The asterisked term A* or B* includes the slow start effect by the rule (e),which has been given in [32] in a single lane model. We can check that the sum of eqs.(3) and (4) is equivalent to eq.(1) by using eq.(2). The simplest way to check this fact is that we take up the possible set of values of A and B and compare the U_{jt} in both equations for all cases. We have confirmed this numerically.

3. Metastable Branches in Fundamental Diagram

In this section, two-lane traffic flow is simulated according to the new model obtained in the previous section, and the fundamental diagram of EBCA1 is discussed by using the two-lane model. In the following, we consider a periodic road, or a circuit. Total flow q_j^t of two lanes is expressed in a conservation form such as

$$\triangle_t U_j^t + \triangle_j q_j^t = 0, \qquad (7)$$

where $\triangle_t$ and $\triangle_j$ are forward difference operators with respect to the indicated variable. The average total flow Q^t over all sites are defined by

$$Q^t \equiv 1/KL \Sigma q_j^t, \qquad (8)$$

where K is number of sites in the periodic road. The average flow of A-lane and B-lane, Q_A^t and Q_B^t can similarly be obtained by flow q_{Aj}^t and q_{Bj}^t of A-lane and B-lane as

$$Q_A^t \equiv 1/K \Sigma q_{Aj}^t, \quad Q_B^t \equiv 1/K \Sigma q_{Bj}^t, \qquad (9)$$

$$Q^t \equiv Q_A^t + Q_B^t . \qquad (10)$$

Figure 2(a) is a density-flow diagram of the total average flow Q^t of EBCA1 with L=2 and K=24. The flow Q^t is recorded during longer time than the period K starting from different initial distributions at various car densities ρ (=N/KL: N is the number of cars). We obtain a unique value of Q^t under the critical density ρ_c

(=1/3). Above ρ_c, many different values of Q^t are obtained for different initial distributions, and the flow seems to distribute at random in the region. Let us study the region in detail in the followings, which is not considered in the previous papers. Under careful observation, it is found that these multi-value states can be classified into three types. The first one is an asymptotically steady state whose value converges on a constant quickly in time. The second is an oscillating state caused by lane change, whose value varies among the constants appearing in the first case with a period of several time-steps. The third is a fluctuating state whose value varies with extremely long period. According to this classification, the density-flow diagram shown in Fig. 2(a) can be considered as superposition of three diagrams. Diagrams of the first and the third type flow are shown in Fig. 2(b) and 2(c), respectively.

All points in Fig.2(b) are on straight lines and the flow quickly relaxes to a steady state in each case. A branch AO indicates free flow states and the DE steady jamming states. A part AD of the AO and the branch BC are metastable[32]. Apart from this main frame, there are many metastable branches in a parallelogram region BDGF. In the region, there are $K/6-1$ straight branches parallel to BC with an equal interval. The points are sited on these branches and make a two-dimensional lattice. As K increases, the interval between the points decreases and the parallelogram region BDGF is filled with the points quasi-continuously. All these points in it are also metastable.

Figure 2(c) shows the third case of fluctuating states. The values fluctuate in time and distribute in the parallelogram region and around the branch BC. Figure 3 shows variation in time of flows Q^t, Q_A^t and Q_B^t at K=240 and ρ = 0.5396, whose values fluctuate with long period, especially extremely long in Q_A^t and Q_B^t. The fluctuating state exists between two critical densities near 5/12 and 3/4, which are just of points B and C. In the region under ρ = 5/12, the state relaxes to free state, jam or the first type metastable state. Figure 4(a) shows variation of the period T of fluctuation of Q^t for various car densities at K=240. The period just equals K at the upper critical density ρ = 3/4. As for Q_A^t and Q_B^t, their periods are the same and a multiplicative numeral of the period of Q^t : T · n (n:integer), where n=22 in this case. The multiplicity n distributes widely from several to several hundreds for different initial distributions and then the period of Q_A^t and Q_B^t usually becomes considerable longer than K. Figures 4(b) and 4(c) show power spectra I of Q^t and Q_A^t, for an initial distribution, respectively. I is defined by $I=|\Sigma Q^t \exp(2\pi ift/T)|/T$. The corresponding periods are marked by f or f_A in each figure. If the flows Q^t, Q_A^t and Q_B^t can be expressed by equations, they are as follows,

$$Q_A^t=a_1\cdot g(\omega t)+a_2\cdot g(2\omega t)+\ldots+a_n\cdot g(n\omega t)+a_{n+1}\cdot g((n+1)\omega t)+\ldots, \quad (11)$$

$$Q_B^t=b_1\cdot g(\omega t)+b_2\cdot g(2\omega t)+\ldots+b_n\cdot g(n\omega t)+b_{n+1}\cdot g((n+1)\omega t)+\ldots, \quad (12)$$

$$Q^t=a'_n\cdot g(n\omega t)+a'_{2n}\cdot g(2n\omega t)+\ldots, \quad (13)$$

where g is an arbitrary function and a_i, a'_i and b_i are constants. Simulation indicates the following fact. The low frequency terms in eqs.(11) and (12) chancel out each other and disappear in eq.(13). However, high frequency term is totally never canceled out and small fluctuation remains on Q^t (see Fig.(3)), i.e. $a_i = -b_i$ except

$i=n, 2n, \ldots$. Frequency f ($\omega = 2\pi f$) is T/n. The fluctuation is induced by lane-change of cars. In any way, T becomes long to almost infinite as K increases. It suggests that the flow in this fluctuating state is chaotic.

In Fig. 5, time series of the fluctuating flows Q_A^t, Q_B^t at $\rho =$ 0.5396 are plotted separately on the density-flow diagram. The Q_A^t and Q_B^t move in two-dimensional region on the diagram because both flow and density of cars in each lane fluctuate in time with long period. For various densities between $\rho = 5/12$ and 3/4, Q^t, Q_A^t and Q_B^t widely cover on the diagram.

4. Flow and Configuration of Cars on the Two-Lane Road and Lane change

Now let us discuss the configuration of the cars in the points on the sub-blanches for a case of K=24. At the point D in Fig.2(b), which is denoted by D_{00} hereafter, the density ρ is 1/3, and the regular configuration is ...100100100... on A-lane, ...100100100... on B-lane, and then A+B=...200200200... on the total lane. At m-th point D_{m0} from D_{00} on the branch AD, m blocks of "100100" on the A-lane are exchanged for m blocks of "101010" and then the configuration at the center point B (D_{40}) of AD is ...101010101010... on whole area of A-lane, ...100100100100... on B-lane, and ...201110201110... on the total lane. At the point A denoted by D_{80}, $\rho = 1/2$, ...10101010... on both lane and ...20202020... on the total lane. As moving from B to F, the blocks of "100" on the B-lane are replaced by "101" and the configuration at the point F (D_{48}) becomes A=...101010..., B=...101101... and A+B=...202111... on the total lane.

At further points over F to C, the block “101” is exchanged for “111” and the configuration goes to A=...101010..., B=...111111... and A+B=...212121... at the point C ($D_{4,16}$). Generally, the configuration of the point D_{mn} on the two-dimensional lattice in the parallelogram BDGF can be expressed by exchanging m blocks of “100100” for “101010” on the A-lane and n blocks of “100” for “”101” on the B-lane from the basic configuration D_{00}. Of course, because the two lanes are symmetric, the role of A and B lane can be exchanged due to initial configurations. From this point of view, we can understand that this kind of points cannot exist out of the parallelogram BDGF. The configurations described here are most regular examples and in general various other configurations made by exchange or shift of the blocks between A- and B-lanes, which correspond also to the same flow point. For example, another configuration ...111111..., composed of ...101010.. and ...010101...shifted by one site, corresponds to point A (D_{80}) also.

The configuration at the points near D on the branch AD can be imagined as that there exist some clusters of “101010” in the configuration of ...100100.... The block 101010 is unstable, because if a “1” in “101010” block is perturbed to be “110010” and then the corresponding spot in the diagram relaxes to a lower-flow point at the same density[32]. We can imagine the “101010” a compact train of cars moving on expressway with high speed. Controlling this unstable state is really planned as the platoon traveling in intelligent transport systems (ITS) to get high traffic flow on expressway.

In the points on the branches inside the parallelogram BDGF and on the blanch BC, there are many configurations of "21" or "12" in the total lane. The configuration indicates that there is a car that can neither advance on its lane nor go to another lane, i.e. the car only stays at its site. Lane-change process is suppressed and a local congested part arises there. Cars in this state advance on own lane with stop-and-go traffic, which does not globally spread but is local and temporal: Car stops at "21" or "12" and then goes out of it. The stopped car becomes a cause for formation of new "21" or "12" in next time-step and then the number of "21" or "12" is conserved in time and the flow keeps constant. The number of "21" and "12" is decided at initial distribution of cars. Many kinds of initial distributions make two-dimensional lattice of local congested states in this area. In conclusion, cars on these points stop and go on their own lane. If once "22" is broken out from "21" or "12" on the lane by some perturbation, the global stop-and go- flow occurs and the flow goes down the global jamming branch DE. Thus the traffic flow in the local congested state is larger than that in the global jamming state.

As mentioned previously, there are many points fluctuating around the two-dimensional lattice or the straight lines. This fluctuation is caused by the lane-change process. A local congestion on A-lane induces a lane-change to B-lane at this case. This process induces new congestion on B-lane, which induces the next lane-change to A-lane. The concatenate lane-change makes large fluctuation on both average flow $Q_A{}^t$ and $Q_B{}^t$. The sequence of lane-changes continues for long time without coming to a stable flow. Now, we introduce a new quantity C which indicates lane-change rate of a car. It is defined as follows,

$$C = 1/TN\Sigma\ \Sigma\ \{\min(1-A^*_{j-1}{}^t,\ 1-A_j{}^t,\ B^*_{j-1}{}^t,\ B_j{}^t) + \min(A^*_j{}^t,\ A_{j+1}{}^t,\ 1-B^*_j{}^t,\ 1-B_{j+1}{}^t)\}, \quad (14)$$

where the first and second terms in the braces indicate numbers of cars changing from B to A lane and from A to B lane, respectively, which appear in eq.(3). They are summarized for all sites (j=1,2...K) during time T and averaged. C is lane-change rate of a car in unit time. Figure 6 shows variation of C for various car densities at K=240. It can be naturally understood that C becomes zero at both low- and high-critical densities (ρ =5/12 and 3/4) and has a peak between them ($\rho \fallingdotseq 2/3$). The value 5/12 of low-critical density is given by the average of 1/2 and 1/3. The local pattern of "101010"+"100100" on both lanes is frequently observed in the simulation for the fluctuation flow. Though we can't explain the configuration and dynamics of the fluctuating state yet, the densities ρ_A=1/2 and ρ_B=1/3 may be critical densities, to continue lane-change in low density limit. For the high-critical density, ρ =3/4=(1+1/2)/2, which may come from the observed pattern "111111" + "101010".

5. Concluding Discussions

In this paper, a two-lane traffic model equivalent to EBCA1 model is studied. We have obtained evolution equations for each lane. In the flow-density diagram, we found many metastable sub-branches besides the main branches. A kind of flow on the sub-branches is stable and cars advance by local and temporal stop-and go-flow on own lane with no lane-change, though the flow is congested locally. This local congested state may be related to convectively unstable state observed in optimal velocity model with open boundary condition[38]. Second kind of flow is the states oscillating with short period. The third kind of flow is local congested states with frequent lane-change. The flow fluctuates in time and varies around the branches on the diagram. Effect of lane-change on flow of each lane is remarkable at magnitude and the periodic time of fluctuation is chaotically very long.

There are many two-lane traffic models including more realistic lane-change models and considering asymmetric roles of two lanes[21-24]. Compared with them, EBCA1 will be too simple model. Nevertheless, it can provide many kinds of metastable states. The fact may suggest that *slow-start, high-velocity and multiple-lane* models are essential processes to bring metastable states into existence in the traffic model. It is worth notice that EBCA1 is perfectly deterministic, not stochastic. For all that, the EBCA1 shows chaotic fluctuating states.

It is known that the average velocities of cars in traffic flow can be nearly synchronized in different lanes of the expressway[33]. Experimental investigations by Kerner and Rehborn[34-37] show that in synchronized traffic flow cars could move with nearly the same velocities in different lanes near on-ramps. They say that there are three phases: free flow, synchronized flow and wide moving jam, in the traffic flow. The jams emerge in free flow through a sequence of two phase transitions: " free flow → synchronized flow → jams". They moreover say that the synchronized flow can further be characterized into three different kinds of phases[35] : 1)both the velocity and the flow are stationary, 2)the velocity is only stationary and the flow is not stationary and 3)both the velocity and the flow abruptly change. Thus the synchronized phase seems to be a key word to clarify the traffic flow, though it has not been fully established. Many metastable states have been found in EBCA1 model and it has been showed that the stable flow in linear branches has constant velocity and flow. This flow may correspond to above type 1) synchronized flow. The type 2) and 3) synchronized flows may be found in the present fluctuating flows. Especially, we think that type 3) has deep relation with our fluctuating flow. In further studies, the local and dynamical configuration of cars in those flows should be studied and behaviors of individual car should be clarified in connection with the synchronized states.

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Figure Captions

Fig.1 Two-lane road and an example of moving of cars. Figures a) and b) show procedure a) and b), respectively.

Fig.2. (a) Density-flow diagram for the total average flow Q^t at L=2 and K=24. (b): Density-flow diagram for the stable flow. (c): Density-flow diagram for the fluctuating flow.

Fig.3 Time variation of the flow $Q_A{}^t$ of A-lane and $Q_B{}^t$ of B-lane in the fluctuating flow and the total flow Q^t.

Fig.4 (a) Period of fluctuation of the total flow Q^t. (b) Power spectra of fluctuation of Q^t. (c) Power spectra of fluctuation of $Q_A{}^t$.

Fig.5 Tracks of fluctuating flows $Q_A{}^t$, $Q_B{}^t$ at $\rho = 0.5396$ plotted on the density-flow diagram.

Fig.6 Lane change rate for a car in the fluctuating flow at various densities.

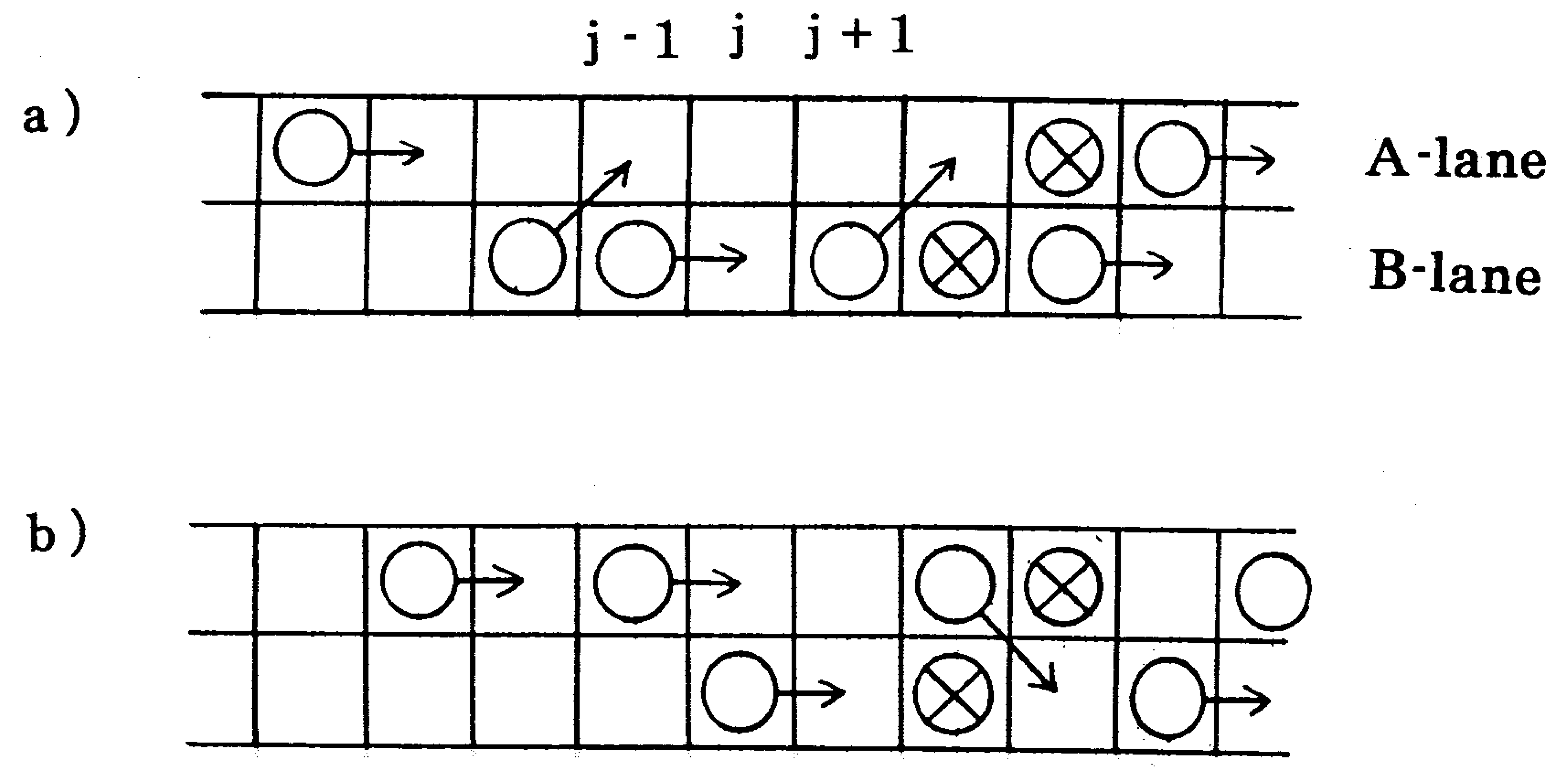

Fig. 1

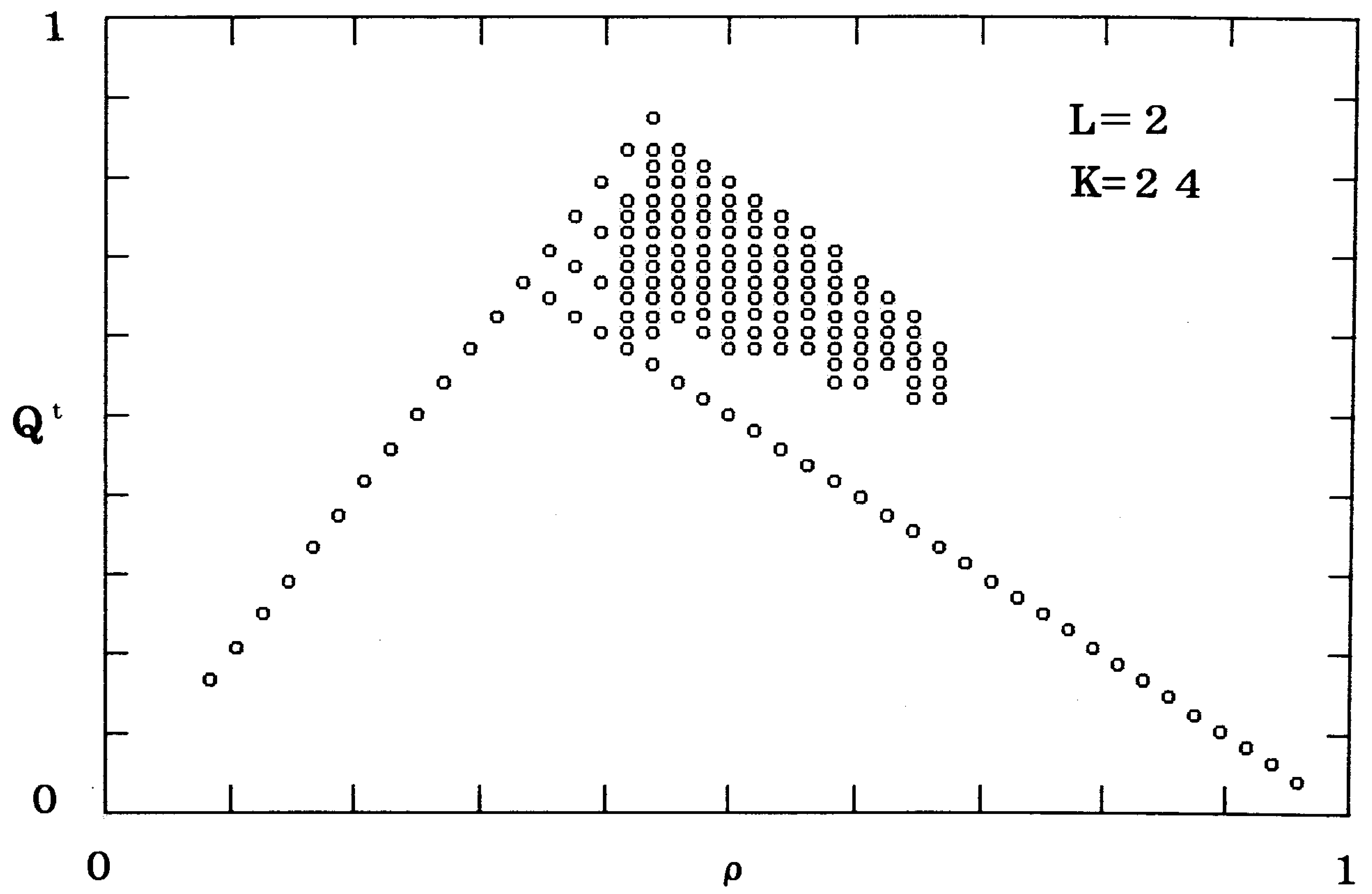

Fig. 2 (a)

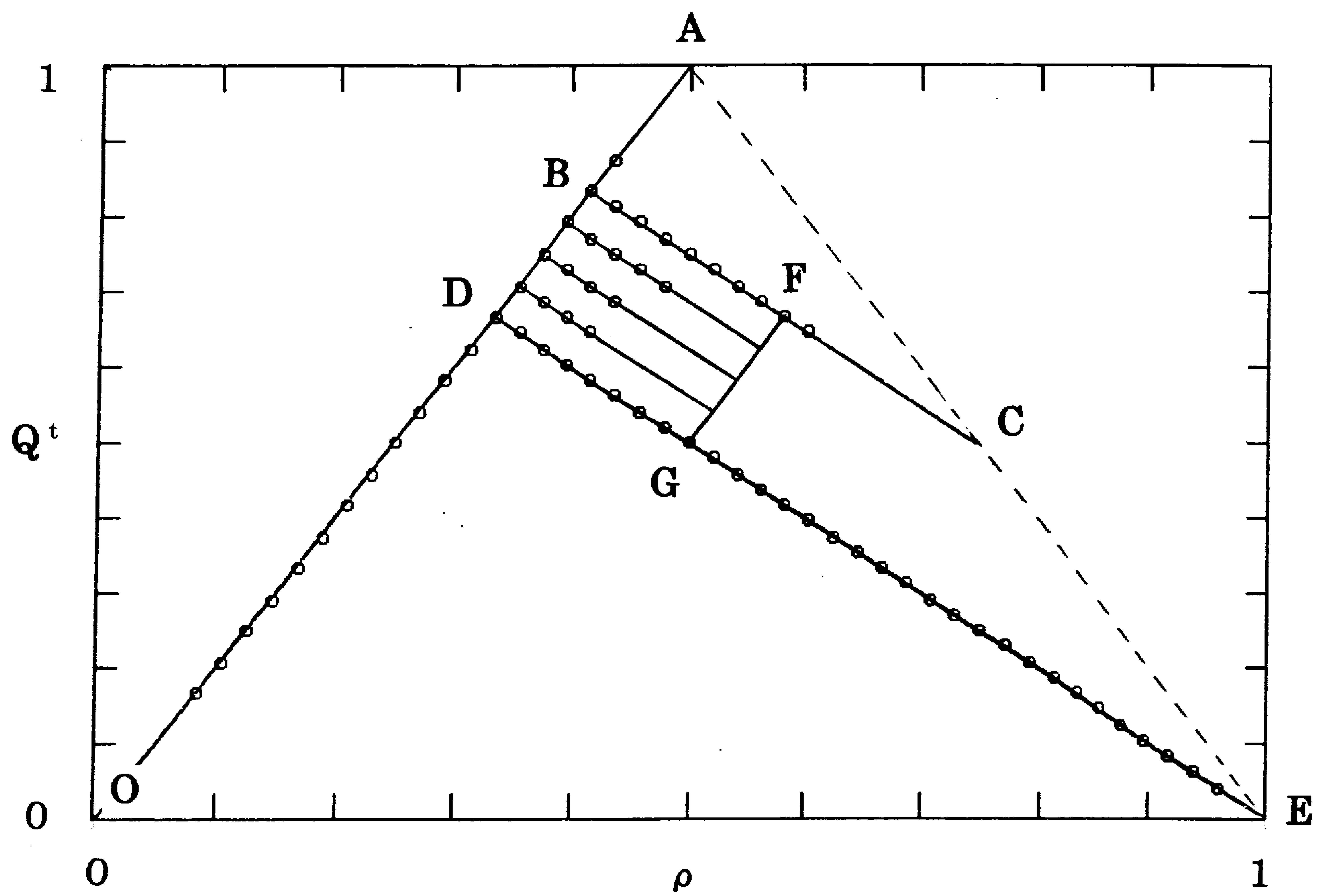

Fig. 2 (b)

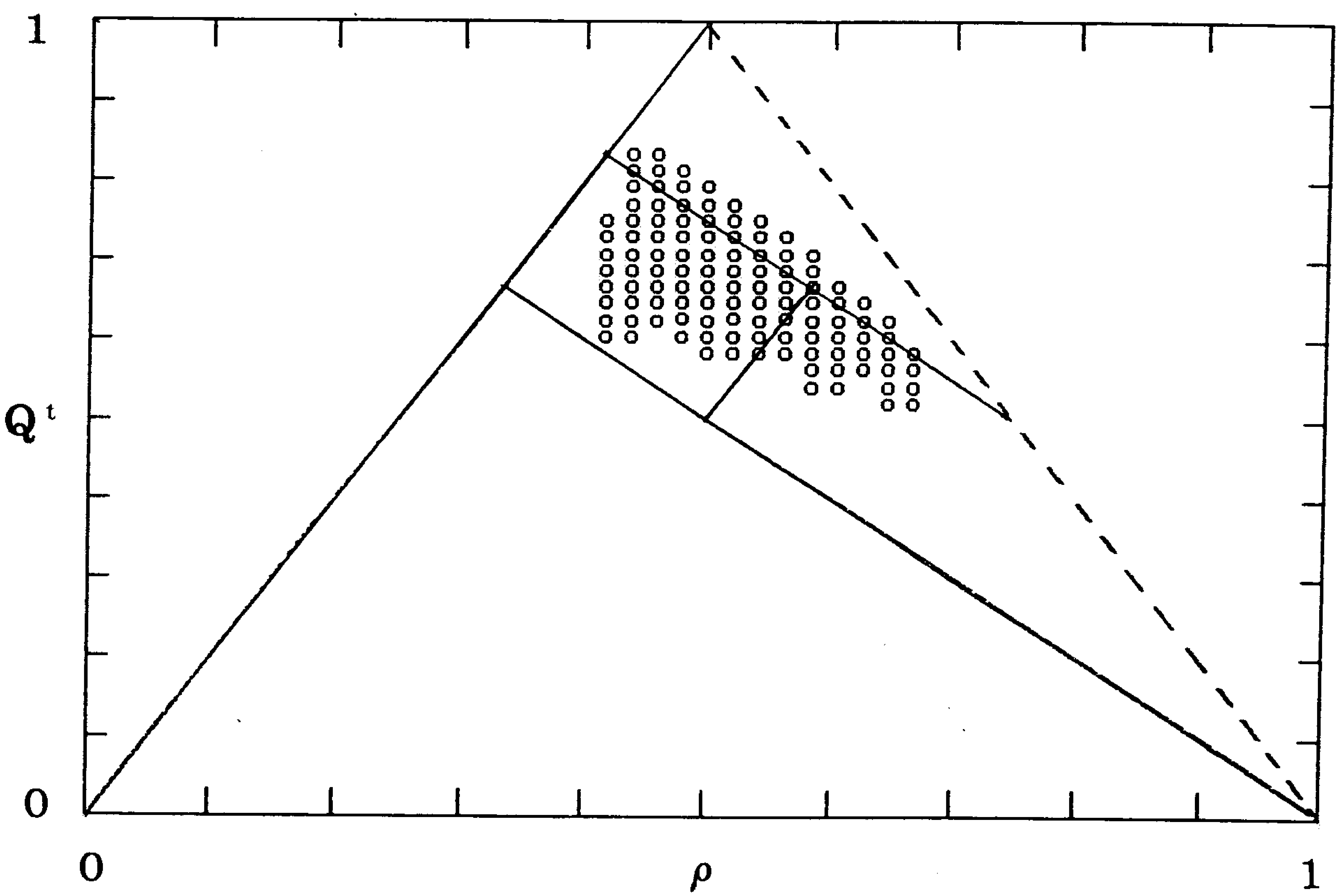

Fig. 2(c)

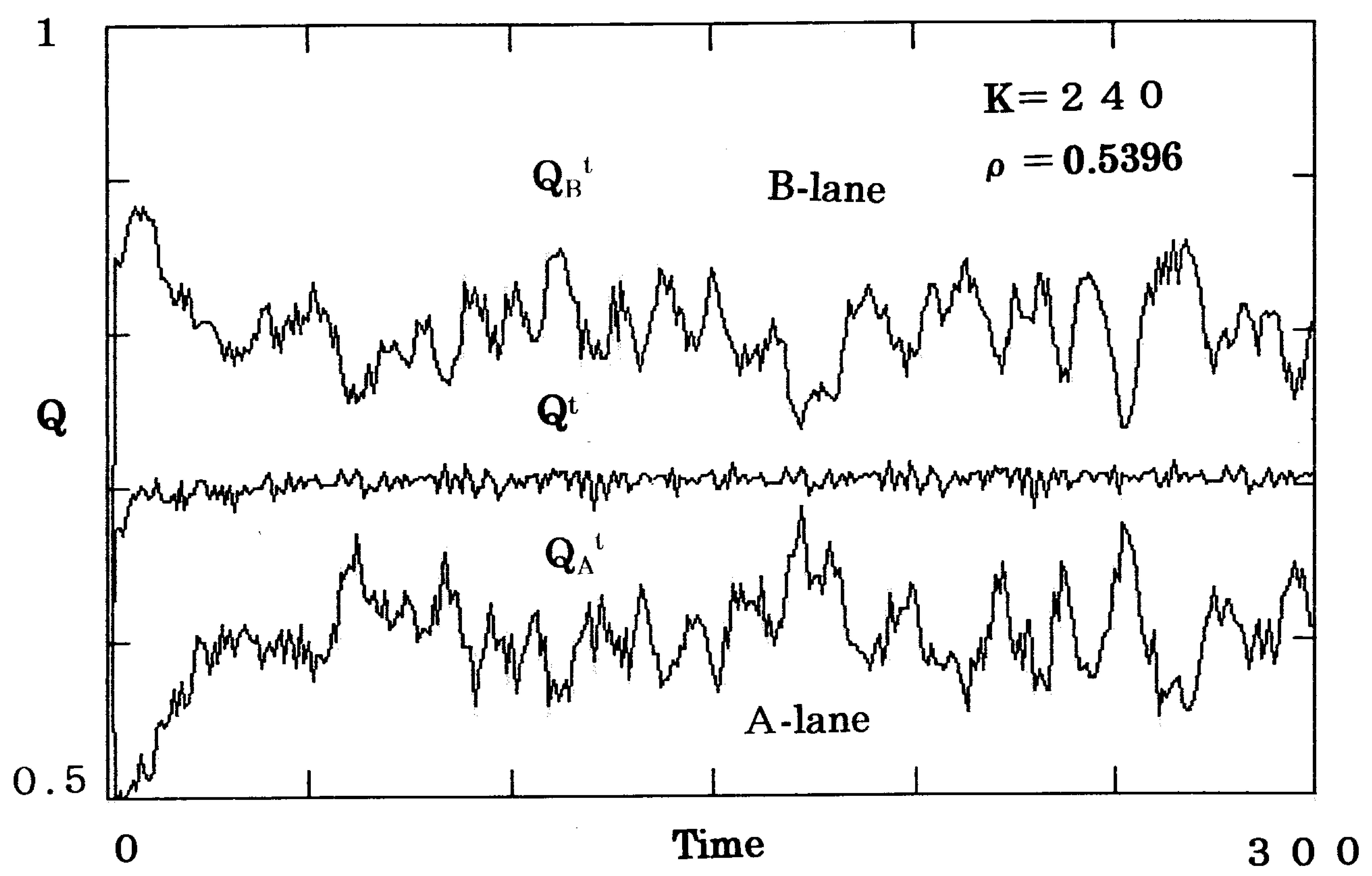

Fig. 3

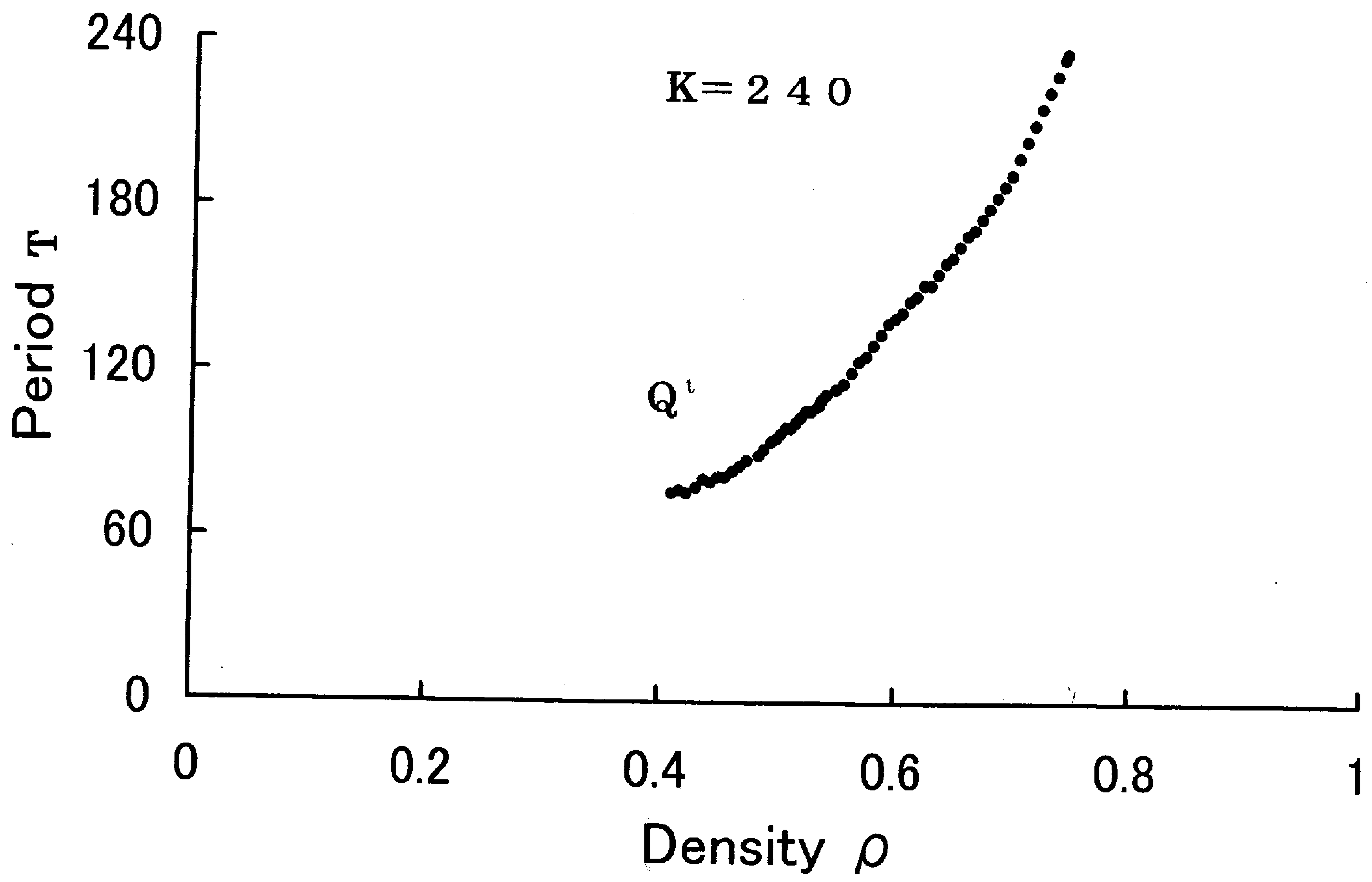

Fig. 4(a)

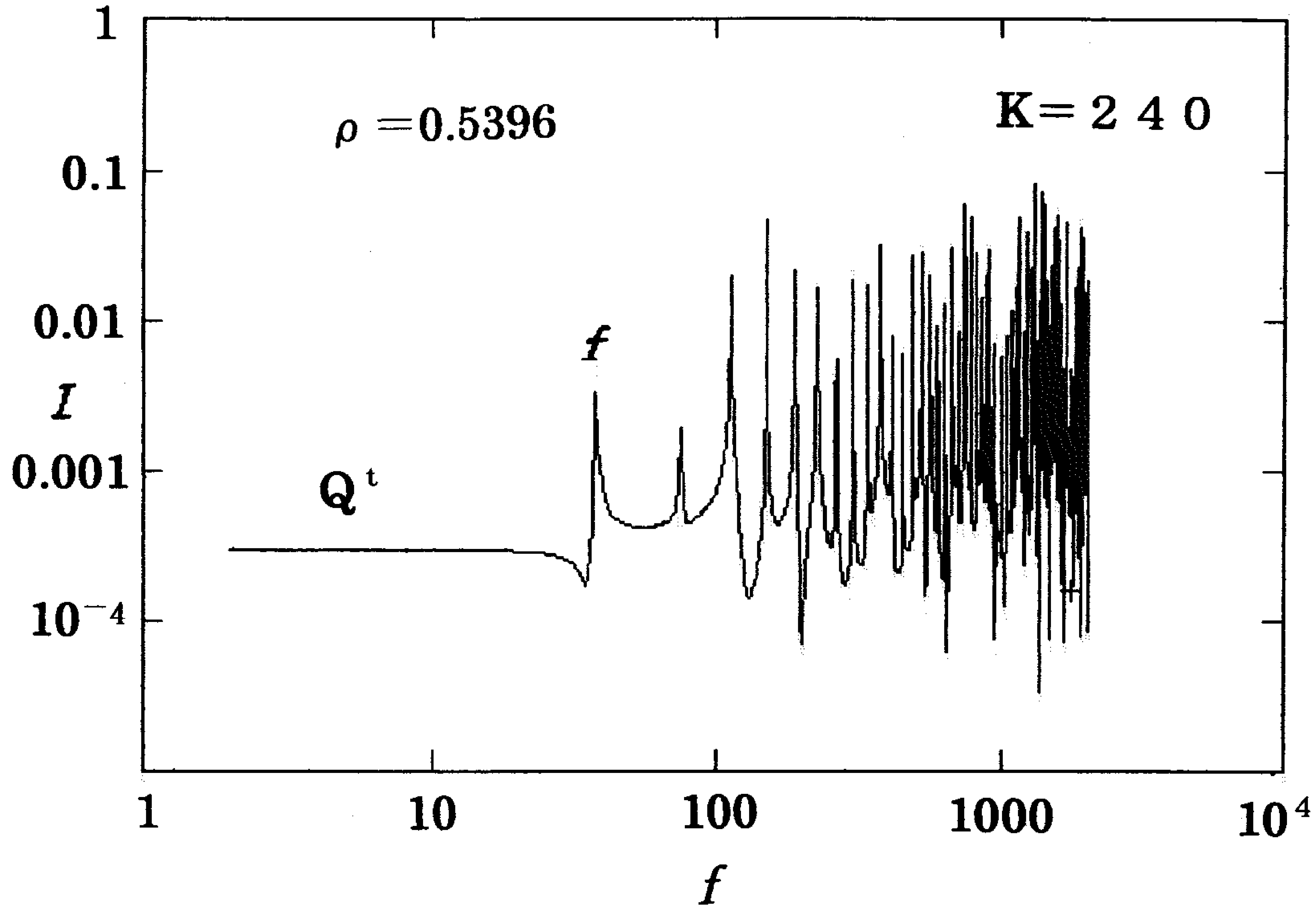

Fig. 4 (b)

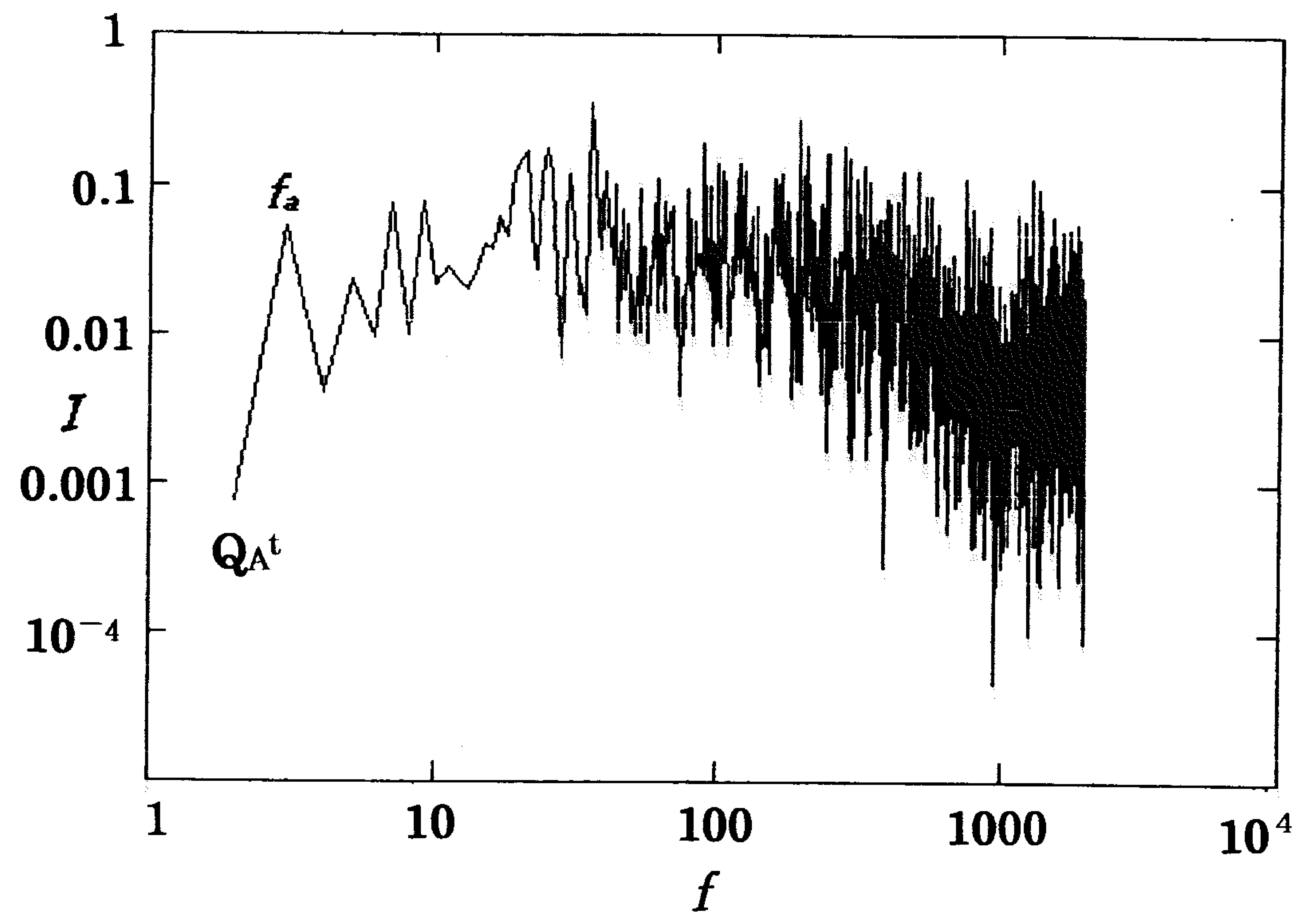

Fig. 4 (c)

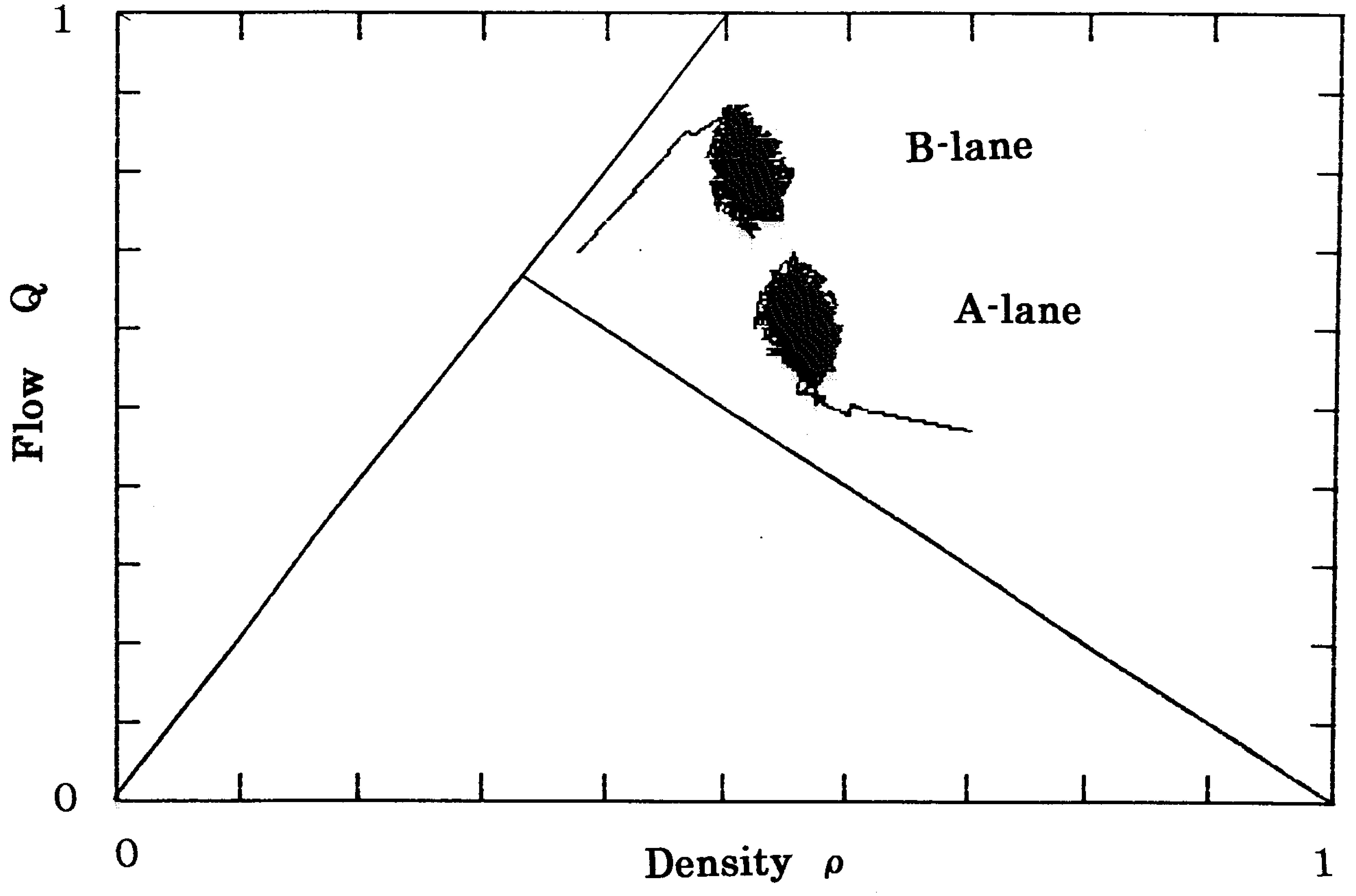

Fig. 5

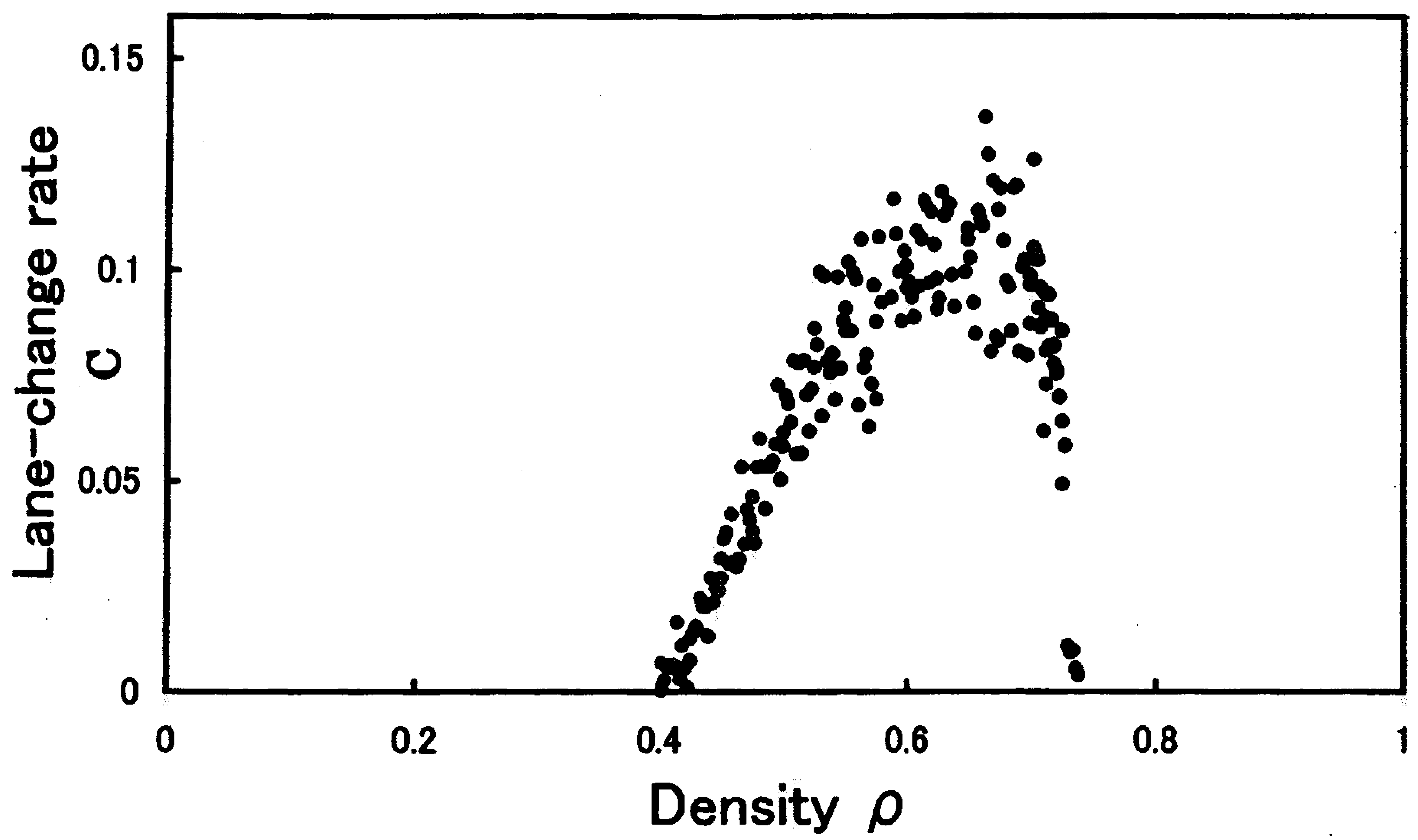

Fig. 6